FELIPE DE FIGUEIREDO SILVA

TRADEOFF BETWEEN AGRICULTURE AND FOREST PRESERVATION IN THE BRAZILIAN AMAZON

Tese apresentada à Universidade Federal de Viçosa, como parte das exigências do Programa de Pós-graduação em Economia Aplicada, para obtenção do título de *Doctor Scientiae*.

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BIOGRAPHY

Felipe de Figueiredo Silva, son of Celso Luis Guimaraes da Silva and Maria Cristina Vieira de Figueiredo Silva, was born on August 3rd of 1985 in Salvador, Bahia.

In October 2005, he began studying Economics at the undergraduate program in the Universidade Estadual de Feira de Santana, from which he graduated in February of 2010.

Felipe initiated his master’s degree in Applied Economics at the Universidade Federal de Vicosa in February of 2010. He defended his thesis, “Market structure and innovation in the seed production of corn, soybeans and cotton in Brazil”, on February of 2012.

Also in 2012, Felipe ingressed on a D. Sc. program in Applied Economics at the Universidade Federal de Vicosa. In August 2013, he also started a PhD program in Agricultural Economics at the University of Nebraska-Lincoln. In February of 2017, he defended his thesis, “Tradeoff between agriculture and forest preservation in the Brazilian amazon”.

SUMMARY

ABSTRACT

SILVA, Felipe de Figueiredo. D. Sc., Universidade Federal de Viçosa, February, 2017. **Tradeoff between agriculture and forest preservation in the Brazilian Amazon.** Advisor: Marcelo Jose Braga. Co-Advisors: Richard K. Perrin and Lilyan E. Fulginiti.

The Brazilian Amazon forest region has experienced agricultural area expansion with high rates of progressive technical change as well as deforestation over the last decade. This has generated concern about the tradeoff between forest preservation and production of agricultural commodities. The literature suggests specifically grains, livestock and timber as the main drivers of deforestation in this region. In this study we estimate the tradeoff between agriculture and forest preservation in municipalities of the Legal Amazon for 2006 and their rate and biases in technical change during the period 2003-2015. To obtain these estimates, we use a directional distance function to estimate a production possibility frontier, considering deforestation as an undesirable output. Using this information we calculate the shadow price of reducing deforestation in terms of agricultural income foregone. Results indicate that, to preserve an average hectare of forest, US$ 796.81 in annual agricultural GDP has to be foregone. At a social discount rate of 10% and a conservative estimate of 100 tons of carbon per hectare of forest, these results imply an average shadow price of US$21.71 per ton of CO_2 emissions. This estimate varies with assumptions on discount rate, carbon content and length of period considered. We have also estimated an average rate of technical change of about 4.6% per year during the period 2003-2015. It means that, with no change in inputs, technical change allowed an expansion of agricultural outputs and a contraction of deforestation of 4.6 % during this period. Technical change has been biased toward agricultural outputs and against deforestation suggesting that increases in output are now possible with less deforestation.

RESUMO

SILVA, Felipe de Figueiredo. D. Sc., Universidade Federal de Viçosa, fevereiro de 2017. ***Tradeoff*** **entre agricultura e a preservação da floresta na Amazônia Brasileira.** Orientador: Marcelo Jose Braga. Coorientadores: Richard K. Perrin e Lilyan E. Fulginiti.

A fronteira agrícola no norte do Brasil tem ocasionado altas taxas de desmatamento na Floresta Amazônica, o que tem gerado debate sobre o *tradeoff* entre a preservação da floresta e a produção agrícola. Segundo a literatura econômica, a produção de grãos, de gado e de madeira são os principais causadores do desmatamento. Nesse estudo, o *tradeoff* entre agricultura e a preservação da floresta é estimado para os municípios da Amazônia Legal para o ano de 2006. A taxa de progresso tecnológico também é estimada para esses municípios para o período 2003-2015. A *output directional distance function* é usada para a estimar a fronteira de possibilidade de produção considerando desmatamento como um produto indesejável. O custo de oportunidade, *tradeoff*, é medido em termos da renda agrícola que tem que se abrir mão para preservar um hectare de floresta. Os resultados mostraram que, para preservar um hectare de floresta Amazônica, os municípios, anualmente, deveriam abrir mão de cerca de US$ 800,00. Medidas de conversão entre 01 hectare de floresta e a quantidade de carbono sequestrada pela floresta permitiram estimar que o custo de manter sequestrada 01 tonelada de CO_2 na floresta são, pelo menos, US$ 21.71. Os resultados indicaram, ainda, que houve progresso tecnológico, na média de 4,6, e que esse foi direcionado a um aumento da produção agrícola, incorrendo em menor acréscimo de desmatamento. Isso significa que, anualmente, a produção agrícola expandiu 4.6%, enquanto o desmatamento contraiu 4.6%.

INTRODUCTION

Agricultural expansion and deforestation are two of the major issues in worldwide climate change discussions due to their impacts on atmospheric Carbon Dioxide (CO_2). Brazil is responsible for the largest tropical forest in the world, the Amazon forest, which accounts for 13% of the world's forest area and 58% of Brazil's surface (Food and Agricultural Organization of the United Nations, FAO, 2015). Strong agricultural expansion, starting in the 1990s, has impacted this area greatly (Nepstad et. al. 2007). Grains, livestock and timber production have been pointed as the main drivers of deforestation in this region (Riveiro *et al.* (2009); Margulis (2004); and Nepstad *et al.* (2001)).

There is a vast literature discussing the Amazon Forest deforestation seeking to identify the main agricultural drivers of deforestation [Riveiro *et al.* (2009); Margulis (2004); and Nepstad *et al.* (2001)], to calculate the opportunity cost of forest preservation [i.e. Nepstad *et al.* (2007), Boner *et al.* (201), May *et al.* (2011), and Heres *et al.* (2013)], and to evaluate the effect of technical change in deforestation [Villoria *et al.*(2014) and indirectly Filho *et at.* (2015)]. None of these studies have treated deforestation as undesirable output directly on the evaluation of these issues. Most of these studies are qualitative or derive the tradeoff between agriculture and forest preservation based on budget data or simulation. In this work, we aim to provide more information by explicitly assuming deforestation as undesirable output when identifying the opportunity cost of forest preservation and the effects of technical change on this tradeoff.

Preserving a hectare of Amazon forest brings many benefits enjoyed by mankind as a whole, while the opportunity costs of preservation fall upon the Brazilians themselves, who must trade off potential agricultural income forgone in exchange for a hectare of forest preserved. We model the tradeoff between forest preservation and agricultural production using a directional distance function to estimate a production possibility frontier introducing deforestation as an undesirable output. This frontier is then used to answer the following questions:

a) What is the opportunity cost of preserving the forest in terms of agricultural output?
b) What is the rate of technical change in agriculture considering deforestation as an undesirable output?
c) Has technical change been biased toward agricultural outputs or deforestation?

To answer the first question, we estimate marginal rates of transformation to identify the opportunity cost of forest preservation in terms of foregone agricultural production. Using coefficients of forest carbon content, we then calculate the opportunity cost of sequestering CO_2 in the forest. To answer the second question, we estimate the shift in the production possibilities frontier due to exogenous technical change in the direction of output expansion and deforestation contraction. To answer the third question we estimate the change in the marginal rate of transformation due to exogenous technical change.

The production possibilities frontier is estimated using different methodologies. In chapter 1 we use Data Envelopment Analysis (DEA), a non-stochastic, non-parametric approach, to identify the opportunity cost of forest preservation in terms of grains, timber and livestock revenues forgone. We use data for 156 municipalities in the "arc of the deforestation" region of the Brazilian Amazon during 2006. In chapter 2, we use a parametric stochastic frontier approach to identify the opportunity cost of forest preservation in terms of agricultural Gross Domestic Product for 590 municipalities in the Legal Amazon forest region of Brazil in 2006. In chapter 3 we use a stochastic frontier approach and data for 200 municipalities in the "arc of deforestation" during the period 2003-2015 to investigate the nature of technical change, that is its rate and bias.

Our study finds higher costs of forest preservation than those in the literature, including the costs used by the Amazon Fund to raise funds to preserving the forest. We also find that during 2003-2015 average productivity of the sector has increased allowing more output produced per unit of deforestation. We note that our results also imply higher abatement costs as a result of potential regulation to reduce deforestation.

CHAPTER 1

TRADEOFF BETWEEN AGRICULTURE AND FOREST PRESERVATION IN THE “ARC OF DEFORESTATION” OF THE BRAZILIAN AMAZON.

1.1.INTRODUCTION

Brazil has the largest tropical forest in world, which is extremely relevant to worldwide biodiversity and the CO_2 cycle. The Brazilian Amazon forest is distributed along the northern states of the country, where deforestation has been affected by agriculture (Hargrave and Kis-Kato, 2013). In 2006, 29% of livestock, 21% of grains and 81% of legal timber production in Brazil was generated in this region (Brazilian Institute of Geography and Statistics – IBGE, 2016). In addition, the National Institute for Space Research (INPE, 2014) indicates that more than six million hectares were deforested in this region during the period between 2004 and 2006. In this paper, we estimate the tradeoff between forest preservation and agriculture by estimating the production possibility frontier (PPF), which also allows us to infer a shadow price of Carbon Dioxide (CO_2).

The economic literature on Amazon deforestation suggests a link between agricultural activities and deforestation [Cattaneo (2001), Morton *et al.* (2006), Rivero *et al.* (2009), Richards et. al. (2012), Hargrave and Kis-Kato (2013), and Richards *et al.* (2014)]. Additionally, a few studies have evaluated the role of Brazil’s participation in REDD+ such as Nepstad *et al.* (2007), Boner *et al.* (2010), May *et al.* (2011), and Heres *et al.* (2013), also taking into account livestock, timber and soybean production activities.

Our study provides an alternative approach for estimating the tradeoff between agricultural production and deforestation. Our approach diverges from these studies by modeling the relation between agriculture and deforestation based on a simultaneous production of desirable (agriculture) and undesirable (deforestation) outputs represented by an estimated PPF. Our focus is on identifying the trade-off between forest and agricultural activities for the agricultural frontier in the Northern region of Brazil or the “arc of deforestation” using municipal scale data from the Agricultural Census of 2006. Thus, we estimate several directional distance functions using Data Envelopment Analysis (DEA), based on Färe *et al.*

(2007). We are not aware of any study that investigates Amazon deforestation using these methods.

On average, to reduce one hectare of deforestation we estimated a revenue foregone of more than nine hundred dollars (from livestock, grains and timber). This implies an average shadow price of a ton of Carbon Dioxide (per ton of CO_2) of US$ 25.00, depending on the discount rates used and the estimates of carbon content on a hectare of forest.

1.2.BACKGROUND

Agricultural expansion in Brazil in the last three decades has driven Amazon deforestation in the Northern region of Brazil. This region is known as agricultural frontier or "arc of deforestation", which represents municipalities with high level of deforestation driven by agriculture. In this paper, we focus on a sample of municipalities in this region with more than 10 thousand hectares of deforestation during the period ranging from 2004-2006. It includes municipalities from the states of Acre (AC), Amazônia (AM), Rondônia (RO), Para (PA), Mato Grosso (MT), Tocantins (TO) and Maranhão (MA). In 2006, more than 1 million hectares[1] were deforested in the Brazilian Amazon, in which 83% was on the municipalities located in the arc of deforestation. See Figure 1.

The total revenue obtained from cattle, grains and timber activities as well as deforestation activities are clustered in the outer boundary of the arc of deforestation, which validates the assertion that deforestation in this region is a by-product of agricultural activity.

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[1] Around 10 thousand km^2.

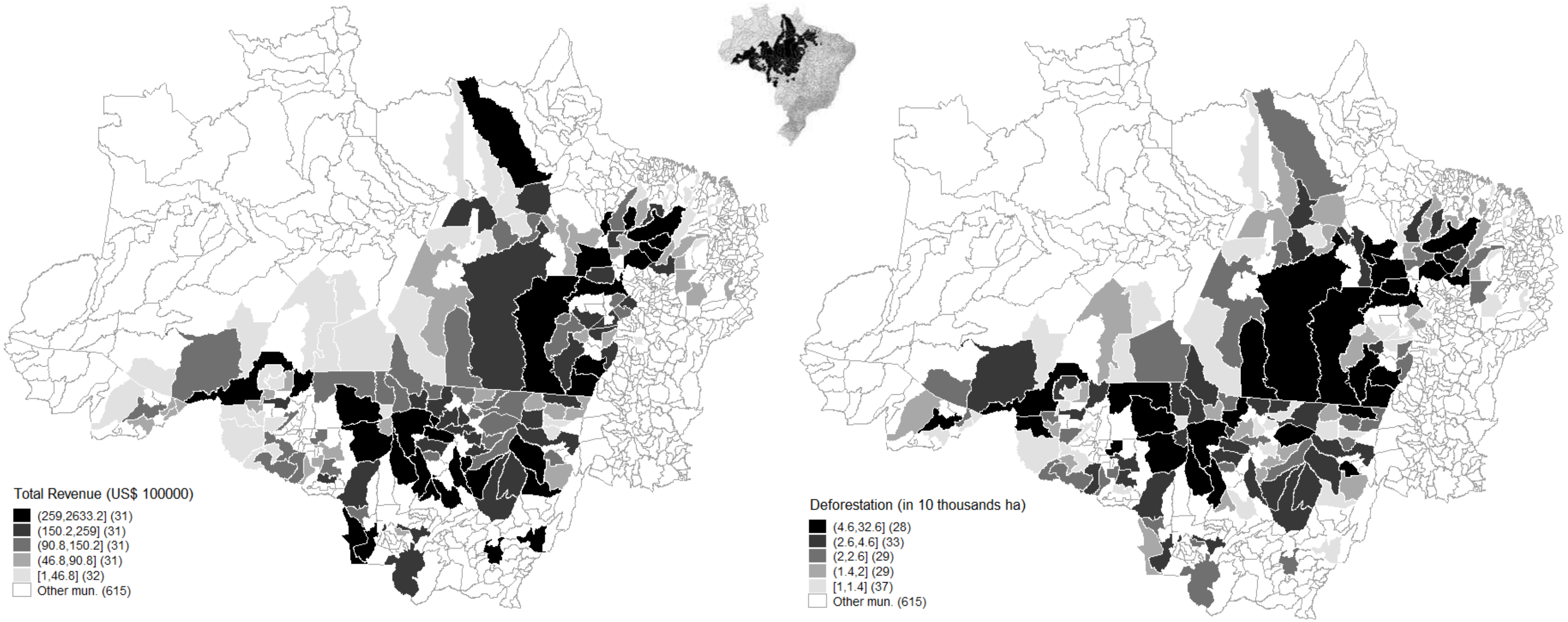

Figure 1 – Total Revenue (in US$ 100 thousand) and deforestation (in 10 thousand hectares)

Note: “Other mun.” in these maps represent municipalities that are not being using on the estimation of Eq. (3).

Source: Own elaboration using Stata 14.

One of the main activities in this region, livestock, is considered as the main driver of deforestation by studies such as Cardille *et al.* (2003), Margulis (2004) and Riveiro *et al.* (2009). The former also suggests crops production. Nepstad *et al.* (2001) and Quintanilha and Lee Ho (2005) indicated timber activity as an important driver of deforestation in this region. Margulis (2004) highlights the dynamic interaction between deforestation and agricultural activities in this region, suggesting a non-contemporaneous relation between them. Additionally, Nepstad *et al.* (2014), Gibbs *et al.* (2014), Hargrave and Kis-Katos (2013) and Soares-Filho *et al.* (2014) suggest that governmental institutions has been contributing to deforestation control[2].

Margulis (2004) estimated a social cost of deforestation taking account not only timber logged but also existence value and *ecotourism*. He found a social cost of US$ 108 per hectare/year. There is a wide range of opportunity cost for a ton of Carbon Dioxide (CO_2) in the literature. Studies that considered Brazil (such as when estimating a price for Latin America) found prices ranging from US$ 1.00 to US$ 50.00 per ton of CO_2. Myers (2008) presents a summary of a ton of CO_2 price for different countries (his Table 2.3). Nepstad *et al.* (2007) estimated the opportunity cost of forest conservation over 30 years and found a value of US$ 5.50. They used a spatial dynamic model mostly based on simulation to obtain an opportunity cost of forest preservation in terms of CO_2 in which livestock, soy and timber production are considered.

Vera-Diaz and Schwartzman (2005) used investment tools and secondary data from the literature to set up a budget and estimate a break-even price for CO_2. They report a total revenue of US$ 3,465 per hectare from one-time logging (US$ 1,435 per ha at a timber potential of $40m^3$/ha), a 30-year present value cattle of ranching of US$106 per ha and a 30-year present value of soybean production of US$ 1925 per ha. They found a break-even price of US$ 22 per ton of carbon and 6.1 per ton of CO_2 (ranges from US$ 3.90 to US$ 6.10 depending on the model used) using a 10% discount rate and an average carbon content of 155 tons of carbon per hectare. Börner *et al.* (2010) used a similar approach to Vera-Diaz and Schwartzman (2005) and found similar results.

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[2] However, Soares-Filho *et al.* (2014) suggested that the Forest Code fails to regulate deforestation on other biomas, such as the Cerrado and Caatinga.

1.3.THE MODEL

We are proposing to use a directional distance function as in Färe *et al.* (2007)[3] to evaluate the trade-off between forest preservation and agricultural production. The output vector with two sub-vectors (***y*, *b***) with $\boldsymbol{y} \in R_+^m$ and $\boldsymbol{b} \in R_{\geq 0}^j$ where *y* denotes the desirable output *y* and *b* the undesirable output; and *x* will be the input vector, $\boldsymbol{x} \in R_+^n$. The undesirable output is deforestation while desirable outputs are livestock, grains and timber production. The output correspondence represents the production technology as

$$P(x) = \{(y, b): x\ can\ produce\ (y, b)\}, x\ \epsilon\ \Re_+^N, \boldsymbol{y}\ \in R_+^m\ and\ \boldsymbol{b}\ \in R_{\geq 0}^j \qquad (1)$$

or the set of desirable (*y*) and undesirable (*b*) outputs that can be produce by inputs *x*, $P(x)$. As in Färe *et al.* (2007), the output set is compact, desirable outputs are strongly disposable, and undesirable and desirable outputs are jointly weakly disposable. This output set is represented in Figure 2, where null-jointness property imposes that undesirable output is a byproduct of the production of desirable output (if $b = 0$, then $y = 0$)[4]. The straight line DE illustrates the strong disposability of desirable outputs while 0BC segment represent weak disposability of undesirable output and null-jointness property.

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[3] Färe *et al.* (1989) modeled a similar issue using hyperbolic measures. Färe *et al.* (2005) and Färe *et al.* (2006) also used directional distance functions to address similar issue. Several other papers used this methodology, see Zhou *et al.* (2014) for a literature review.

[4] Macpherson *et al.* (2010) discuss these properties and its suitability to the estimation of a similar topic.

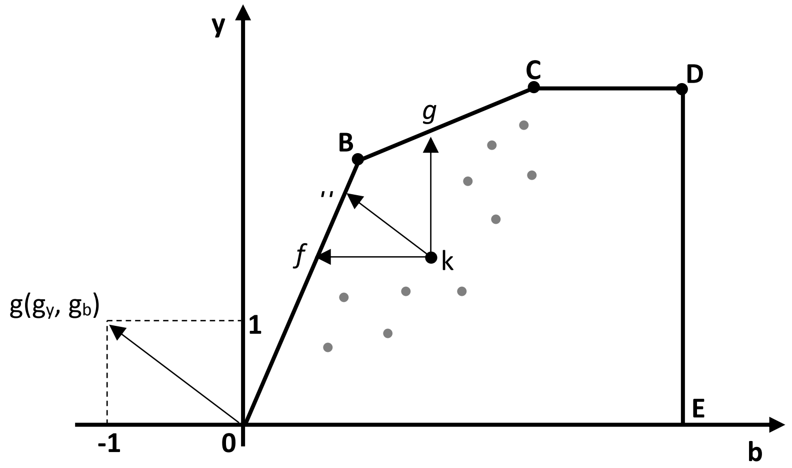

Figure 2: Output Set - *P(**x**)*, and directional output distance measures

We identify the distance to the output set using several different directional distance functions, the directional distance function is

$$\vec{D}_j\left(\boldsymbol{x},\boldsymbol{y},\boldsymbol{b}; g_y, -g_b\right) = max\left\{\beta: \left(\boldsymbol{y} + \beta g_y, \boldsymbol{b} - \beta g_b\right) \epsilon\ P(\boldsymbol{x})\right\} \tag{2}$$

where the pre-determined directional vector is defined as $\boldsymbol{g} = (\boldsymbol{g}_y, \boldsymbol{g}_b)$, and the subscript j refers to output directional distance functions with diferent directional vectors. Three directional vectors relevant to this study are illustrated in Figure 2, $g - (1{,}0)$, $g = (1, -1)$ and $g = (0, -1)$. The vector $g = (1{,}0)$ expands desirable output for given levels of undesirable output and projects the allocation at point k to point g. The vector $g(0, -1)$ contracts undesirable output for given levels of desirable outputs and projects allocation k to f. Finally, $g = (1, -1)$ implies simultaneous expansion of desirable outputs and contraction of undesirable outputs and projects allocation k to point h.

On the boundary 0BC, regulations such as disposal fees and quantity restrictions on undesirable output production impose a tradeoff between desirable and undesirable outputs. This structure implies a positive non-negative marginal rate of transformation between these two types of output.

We use DEA, a nonparametric nonstochastic approach, to calculate the directional distance and the production possibilities frontier. The linear piecewise technology identified by this approach seeks to increase desirable outputs and decrease undesirable output production simultaneously by solving the following problem

$$\vec{D}_j\left(y^k, b^k, x^k; g_y, g_b\right) = \max\left\{\beta^k : \left(y^k + \beta^k g_y, b^k - \beta^k g_b, x^k\right) \epsilon\, P(x^k)\right\} = \beta^{k*} \quad (3)$$

Subject to

$$\sum_{k=1}^{K} z^k y_1^k \geq y_1^k + \beta^k g_1,$$

$$\sum_{k=1}^{K} z^k y_2^k \geq y_2^k + \beta^k g_2,$$

$$\sum_{k=1}^{K} z^k y_3^k \geq y_3^k + \beta^k g_3,$$

$$\sum_{k=1}^{K} z^k b_1^k = b_1^k - \beta^k g_b,$$

$$x_n^k \geq \sum_{k=1}^{K} z^k{}_k x_n^k, \qquad n = 1, \ldots, N$$

$$z^k \geq 0, \qquad k = 1, \ldots, K$$

where $\beta^{k^*} = \vec{D}_j\left(y^k, b^k, x^k; g_y, g_b\right)$ or the distance of the unit *k* from the frontier (a measure of inefficiency); g_y for *y* = 1,2 and 3 or respectively timber, livestock and grains; $n = 1, \ldots, N$ refers to *n* inputs; weak disposability of undesirable outputs is assumed ($\sum_{k=1}^{K} z^k b_1^k = b_1^k$) to model the output set as in Figure 2; and z^k are the intensity variables, where constant returns to scale is assumed.

First, in Figure 2 a movement upward of the unit *k* (northern region of the output set) is modeled by assuming a directional vector of $g = (g_1, g_2, g_3, g_b) = (1,1,1,0)$, where the objective function seeks to increase desirable output production of unit *k* while keeping the same level for the undesirable output. Technical inefficiency is represented by the distance from each observation *k* to the frontier – i.e. *g* – *k* in Figure 2. This is also presented in Table 1 (second row), which summarizes all technologies estimated in this paper.

Second, a directional output distance function assuming a directional vector of $g = (g_1, g_2, g_3, g_b) = (0,0,0,-1)$ is estimated, where the objective function seeks to decrease undesirable output production of unit *k* keeping the same level of desirable output and inputs. Technical inefficiency is represented by the distance between the observation and the frontier – i.e. *f* – *k* in Figure 2.

All variables are normalized by the mean, so $y_m^k = y_m^{k*}/\bar{y}$ where y_m^{k*} represents the actual observation and $\bar{y}$ represents the overall mean, as in Färe *et al.* (2005). In our case, the maximum desirable output achieved by unit *k* under zero inefficiency (or when the observation is driven to the boundary) is found as $y_m^k + \beta^k * \bar{y}$ for the models where $g_y \neq 0$.

Table 1 – Models with alternative directional vectors, $g(g_1, g_2, g_3, g_b)$, in Equation (3)

Models	Directional vectors	Brief model explanation
A	$g = (1,1,1,-1)$	Simultaneous expansion of all desirable outputs and contraction of deforestation
B	$g = (1,1,1,0)$	Expansion of all desirable outputs given an amount of deforestation
C	$g = (1,0,0,0)$, $g = (0,1,0,0)$ and $g = (0,0,1,0)$	Expansion of one desirable output given an amount of deforestation
D	$g = (1,0,0,-1)$, $g = (0,1,0,-1)$ and $g = (0,0,1,-1)$	Simultaneous expansion of one desirable output and contraction of deforestation
E	$g = (0,0,0,-1)$	Contraction of deforestation given amount of all desirable outputs production

Note: $(g_1, g_2, g_3, g_b) = (g_{Timber}, g_{Livestock}, g_{Grains}, g_{Deforestation})$.

The objective of this paper is to estimate the trade-off between agricultural commodities and forest, which can be represented by the difference between points in the frontier boundary. We propose a trade-off measure based on the different quantities achieved by projecting the observed unit *k* to the frontier by two different directions, designated here as *i* and *h*:

$$\Omega_y = \sum_{m=1}^{M} p_m^k * [y_{m,D_i}^k - y_{m,D_h}^k], \quad for\ i \neq h, m = 1, 2, or\ 3\ and$$

$$\Omega_b = b_{1,D_i}^k - b_{1,D_h}^k, \quad for\ i \neq h\ and\ where \tag{4i}$$

$$y_{m,D_i}^k = y_m^k + \bar{y} * \vec{D}_i\left(y^k, b^k, x^k; g_y, g_b\right) * g_y, \qquad i = h = y; b; or\ y, b$$

$$b_{1,D_i}^k = b_1^k + \bar{b} * \vec{D}_i\left(y^k, b^k, x^k; g_y, g_b\right) * g_b, \qquad i = h = y; b; or\ y, b$$

where y_{m,D_i}^{k} represents the potential level of desirable output measured by distance function *i*; b_{1,D_i}^{k} is the potential reduction in bad output using distance function *i*; and p_m^k is the agricultural output price obtained in the Agricultural Census dividing output value of production by its quantity. Distance function *i* projects the observation to the maximum achievable output while holding bad output constant, while distance *j* projects it to the maximum achievable reduction in bad output while holding good output constant. Our measure of tradeoff is the revenue forgone to achieve a reduction in the undesirable output. This we measure as the ratio of the two equations in (4i), Ω_y and Ω_b, the value of agricultural output that must be foregone per unit of deforestation reduced:

$$Tradeoff = \left(\frac{\Omega_y}{\Omega_b}\right), \qquad in \frac{US\$}{hectare}. \tag{4ii}$$

This measure will be calculated using a three step procedure. First, we calculate Ω_y and Ω_b for each observation. Second, we aggregated these measures across states and region. Third, we evaluate Equation (4ii) using the aggregate estimates. It will also be converted in terms of Carbon Dioxide (CO_2) using an estimate of the density of CO_2 per hectare, which in our case comes from the Brazilian Ministry of Environment (MMA/DPCD – 2011). Equation (4ii) represents the revenue foregone in one year as a result of deforestation. To preserve the forest, this amount must be foregone each year, so we also calculate Net Present Value of this stream in perpetuity, which allows us to compare our results with those in the literature. The linear programs described in Equation (3) were estimated using GAMS[5].

1.4.THE APPLICATION

The Legal Amazon consists of more than 700 municipalities stretched in nine states (Amapá – AP, Acre – AC, Amazonas – AM, Mato Grosso – MT, Maranhão – MA, Tocantins – TO, Para – PA, Rondônia - RO and Roraima – RR). The arc of deforestation defined in this paper

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[5] We would like to thank Carl A. Pasurka Jr. for his help sending us the codes for his paper (Färe *et al.*, 2007), which was used as base to build our codes.

consists of 156 municipalities that deforested more than 10 thousand hectares over the period ranging from 2004 to 2006. These municipalities coincide with the agricultural frontier (as you can see in Figure 1), where is constantly pointed out as the main driver of deforestation. This sample of municipalities also has a more homogenous technology, which is highly desirable on DEA estimations[6].

The dataset consists of 156 municipalities obtained in the Agricultural Census of 2006 (online) at *Intistuto Brasileiro de Geografia e Estatistica* (IBGE). Three agricultural commodities are considered desirable outputs – livestock, grains and timber production. The inputs used are labor, capital, area, production expenses such as energy and fuel and cattle inputs. Descriptive statistics are in Table 2. In Appendix 1.A we present descriptive statistics per state.

Deforestation is measured in hectares. Margulis (2004) suggests that deforestation might occur over three years, and be detected only in the third year of the process, depending on the process of deforestation used. Thus, we measure deforestation in 2006 as the average of the previous three years (2004-2006). It was obtained from the PRODES/National Institute for Space Research (INPE, 2014) website. The variable representing timber production was based on Merry *et al.* (2009). They used m^3 of wood production available from IBGE (Table 289) increased by 50% to account for illegal logging not in IBGE. Livestock was measured as number of cattle slaughtered and sold (sold heads), given the importance of livestock in the region. Grains are the sum of soybean and corn production (in tons). The output price for grains used is a weighted average of the prices of both crops where the weights are the relative importance of that crop in amount of production for the municipality. Figure 1 illustrates that higher level of deforestation and of agricultural revenue is clustered in the outer border of the arc of deforestation.

In the input side, labor is the number of employees over 14 years of age, capital is obtained by adding the number of equipment and machinery in the municipality following Bragagnolo *et al.* (2010), and area consists of the total farm area in hectares. Expenses related to fuel and energy (electricity) were aggregated into one variable. Expenses[7] related to seed, pesticides and fertilizers were also aggregated into a single variable to represent other crop inputs. Finally,

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[6] This method suffers more with outliers than the stochastic estimation.

[7] Expenses were obtained from Table 820 on IBGE (SIDRA) website.

expenses related to animal medication, animal purchase and feed were aggregated into one variable to take into account inputs related to livestock. Municipalities in Mato Grosso have higher expenses in these three inputs, which highlights its importance on agricultural production in this region. In the application, all the variables were divided by their means, shown on Table 2.

Table 2 – Descriptive statistics for output and input variables for the municipalities (number of observations = 156) in the arc of deforestation

Variables	Mean	Stand. Dev.	Min.	Max.	Total
Output:					
Livestock (Sold Heads)	34838.63	31004.38	355	167495	5434827
Grains (tons)	43241.38	188254.60	0	1998855	6745655
Timber (m^3)	95342.97	237423.12	0	2100000	14873504
Average Deforestation (hectares - 2004-06)	10993.72	11111.49	3363.333	108413.3	1715020
Inputs:					
Labor (number of employees)	5465.15	4469.02	490	37360	852564
Capital (units)	2530.87	1604.38	364	11283	394815
Area (hectares)	353969.20	265664.22	28188	1581759	55219195
Fuel (US$ 1000)	$4,554.42	11795.96	0	141351	$327,414.75
Crop. Inputs (US$ 1000)	$20,874.52	104322.94	0	1069961	$1,500,656.68
Livestock inputs (US$ 1000)	$8,837.27	8039.96	117	37226	$635,305.99

Note: Descriptive statistics per states are available upon request from the author. All variables are for 2006 except deforestation which considers the period 2004-2006.

1.5.RESULTS AND DISCUSSION

In this section, we present results for the models described in Table 1 which uses Equation (3) for the estimation of the directional distance functions which consider livestock, grains and timber production as desirable output, deforestation as undesirable output and six inputs. The average distances (inefficiency) per state obtained for selected models are displayed in Table 3.

Table 3– Average distances measured for the models (Table 1) estimated by equation (3)

State/ Region	**Models (described in Table 1)**		
	A	**B**	**E**
Rondônia (RO)	0.24	0.29	0.80
Acre (AC)	0.22	0.27	0.45
Amazonas (AM)	0.09	0.09	0.38
Para (PA)	0.23	0.25	0.91
Tocantins (TO)	0.22	0.39	0.27
Maranhão (MA)	0.15	0.18	0.41
Mato Grosso (MT)	0.24	0.31	0.68
Arc of Deforestation	**0.22**	**0.27**	**0.73**

Note: Model A is represented by $D_{y,b}(y^k, b^k, x^k; 1,1,1,-1)$, B by $D_y(y^k, b^k, x^k; 1,1,1,0)$, E by $D_b(y^k, b^k, x^k; 0,0,0,-1)$.

Source: Own elaboration.

The number of efficient municipalities ($\vec{D_i} = 0$) found for Equation (3), was higher where the directional vector related to desirable outputs is not zero ($g_y \neq 0$). For the models A and B, which considered an expansion of all three desirable outputs, 27% (42 municipalities) of the municipalities were efficient – they established the frontier of the production set. The outcome of model A is displayed in Figure 3. Although MT and PA municipalities have shown the highest number of efficient municipalities (15 or 35% of each state's municipalities), proportionally the state of Amazonas has shown a higher proportion of efficient municipalities (50%) and Roraima a lower proportion (13%). On average, a 22% increase in output and a 22% decrease in deforestation could have been achieved by overcoming inefficiency. By only correcting inefficiency, we would observe a decrease in deforestation of more than two thousand hectares on the average municipality (0.22 x 10994 = 2418.68).

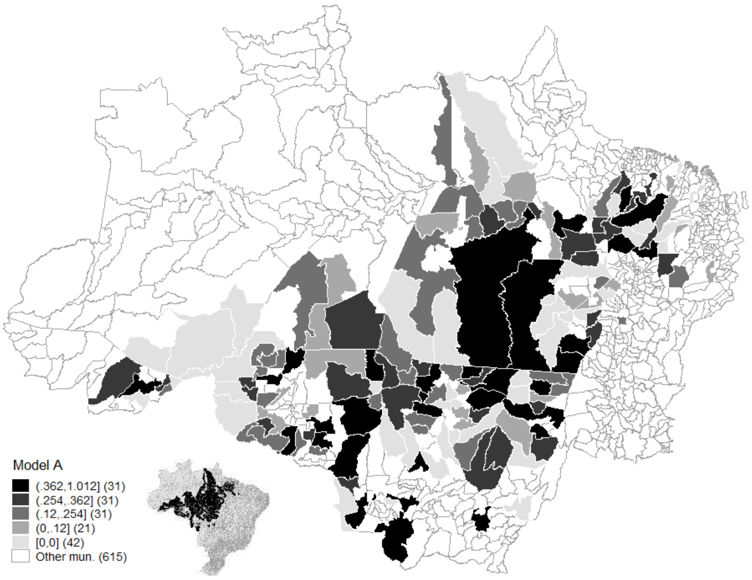

Figure 3 – Geographic distribution of the shadow prices from model A for the Arc of Deforestation (plotted on the Amazon Region map)

Note: Model A is represented by $D_{y,b}(y^k, b^k, x^k; 1,1,1,-1)$, "Other mun." in these maps represent municipalities that are not being using on the estimation of Equation (3).

The main objective of this paper is to evaluate the tradeoff[8] between the value of agricultural commodities and forest. These results are obtained by comparing model B $[g = (1,1,1,0)]$, where an expansion of all three ag. activities occur, and E$[g = (0,0,0,-1)]$, where only a contraction of deforestation occurs. By evaluating Equation (4ii)[9], we found that, on average, US$ 920.61[10] of revenue from cattle, grains and timber has to be foregone to decrease one hectare of deforestation. Tocantins and Mato Grosso have higher shadow prices. The latter state also has the largest agricultural production in the area, which suggests that deforestation

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[8] In Appendix 1.B we present these tradeoff in terms of ag. activity quantity foregone to decrease one hectare of deforestation. Numbers found are higher than observed given that these models impose a one-output reduction (only livestock, grains or timber) to decrease deforestataion.

[9] Our estimates of tradeoff in terms of foregone forest and tons of CO_2 are lower when comparing models B and A $[g(1,1,1,-1)]$. Therefore, the cost of preservation becomes US$ 514.16 per hectare of forest and US$ 1.40 per ton of CO_2.

[10] At an average price of US$ 920.61, a reduction to zero in the amount of deforestation occurred in 2006 would cost more than 50% of the total revenue, from the three agricultural activities used in this study, for the municipalities in the arc of deforestation.

control would have a more severe effect in the revenue of the largest agricultural producer in the region.

Table 4 –Revenue foregone per hectare of forest (deforestation) and per tons of CO_2, obtained using equation (4)

State	Forest (ha) Shadow prices	CO2 Shadow prices
Rondônia (RO)	$812.47	$2.21
Acre (AC)	$1,233.74	$3.36
Amazonas (AM)	$604.82	$1.65
Para (PA)	$796.44	$2.17
Tocantins (TO)	$2,555.44	$6.96
Maranhão (MA)	$1,000.98	$2.73
Mato Grosso (MT)	$1,091.39	$2.97
"Arc of deforestation"	**$920.61**	**$2.51**

Figure 4 shows the geographic pattern of this tradeoff, which illustrates that the outer border of the arc of deforestation has higher shadow prices for sequestering CO_2, reflecting the importance and development of agriculture in this area.

Given our measure of the shadow price for a hectare of forest preservation, it is possible to calculate also the shadow price for sequestering a ton of Carbon Dioxide (CO_2). MMA/DPCD (2011) establishes that one hectare of forest has 100 tons of carbon, and one ton of carbon is equivalent to 3.67 per ton of CO_2. Using this carbon density, our results imply an annual average foregone income of US$ 2.50 per ton of CO_2 (= US$ 920.61/367) To sequester the carbon in perpetuity, this amount must be foregone every year. This is equivalent to a present value of US$ 25.00 per ton when discounted at 10%, US$50.00 per ton at a 5% rate.

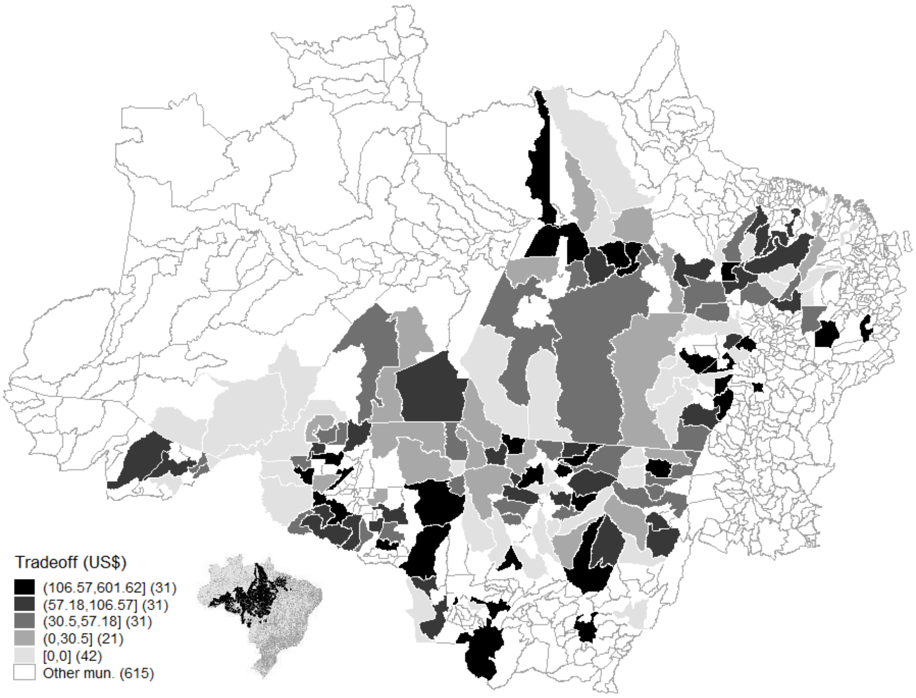

Figure 4 – Tradeoff between agricultural activity (US$) and forest (hectare) – revenue from the three outputs foregone by one less hectare of deforestation – using equations (4)

Note: "Other mun." in these maps represent municipalities that are not being using on this paper.

Source: Own elaboration using Stata 14.

Nepstad *et al.* (2007) estimated an annual opportunity cost of one ton of carbon from timber, livestock and soy of US$5.50 using their 5% discount rate. Vera-Diaz and Schwartzman (2005) estimate annual opportunity cost from cattle, soy and timber of US$6.10 per ton of CO_2 at the 10% rate they used. Our estimate price of a ton of CO_2 is US$12.41 if we calculate it as these authors: using a 30-years NPV analysis for cattle and grains foregone revenue and one-time foregone revenue for timber, using a 10% discount rate and a carbon content of 155 tons of carbon per hectare.

1.6.CONCLUSIONS

Preserving a hectare of Amazon forest brings many benefits. The opportunity costs of preservation fall upon the Brazilians themselves, while mankind as a whole receives the benefits. This paper provides new estimates of the tradeoff between forest and agricultural commodities at a municipal scale for the Legal Amazon, in Brazil. This study contributes to the literature by using DEA to estimate a *Production Possibility Frontier* that includes the relation between agriculture and deforestation.

Overall, we found that more than nine hundred dollars in total revenue from livestock, grains and timber has to be foregone yearly to decrease one hectare of deforestation, implying that the annual cost of sequestering CO_2 by reducing deforestation is about \$2.50 per ton. Thus, the present value of the average shadow price for sequestering CO_2 permanently is \$25 per ton at a 10% discount rate, \$50 per ton at a 5% discount rate. These estimates are several times higher than previous estimates of US\$2.80 to US\$ 6.10 by Nepstad *et al.* (2007), Vera-Diaz and Schwartzman (2005).

This analysis does not attempt to quantify all the benefits of forest preservation, but instead estimates the opportunity cost to Brazil of reducing deforestation in terms of foregone commercial agricultural activity. The study does not consider the impact of changes in expectations and dynamic adjustment that might be important given the period of analysis and the type of products. It would be useful to examine how agents involved might respond to market driven policies given the production possibilities identified here, but that is beyond the scope of the present paper.

CHAPTER 2

TRADEOFF BETWEEN AGRICULTURE AND FOREST PRESERVATION IN THE BRAZILIAN AMAZON

2.1.INTRODUCTION

Agricultural expansion and deforestation are two of the major issues in worldwide climate change discussions due to their impacts on atmospheric CO_2. Brazil is responsible for the largest tropical forest in the world, the Amazon forest, corresponding to 13% of the world's forest area and 58% of Brazil's surface (Food and Agricultural Organization of the United Nations, FAO, 2015). Strong agricultural expansion, starting in the 1990s, has impacted this area greatly leading to the introduction of forest conservation policies (Nepstad et. al. 2007).

This study focuses on a region referred to as the Legal Amazon comprising more than 500 municipalities in nine northern states of Brazil (Brazilian Institute of Geography and Statistics, IBGE, 2015). We estimate the opportunity cost of preserving the forest by finding the marginal rate of transformation between agricultural activities and deforestation. We use a stochastic approach, based on Färe *et al.* (2005) and Färe *et al.* (2006), to fit a quadratic directional output distance function to describe the technological opportunities that includes deforestation as an undesirable output. We are not aware of any other paper that analyzed the cost of Amazon deforestation using an approach similar to this.

We measure opportunity cost of forest preservation in terms of agricultural Gross Domestic Product (GDP) given up. Our analyses contribute to the literature by providing new estimates of the shadow prices of forest preservation and of reduced CO_2 emissions using an alternative approach to those found in the literature. Our results indicate that, on average, US$ 796.81 of agricultural GDP annually is foregone to preserve one hectare of Amazon forest. Given previous estimates of CO_2 sequestered per hectare of forest, we estimate the shadow price of CO_2 to be, on average, US$ 21.70 per ton. 86% of the municipalities have a shadow price lower than this average.

2.2.BACKGROUND

The Legal Amazon refers to an area that includes municipalities in the Amazon Forest biome, in addition to other biomes, in nine northern Brazilian states. It includes municipalities in the states of Acre (AC), Amazônia (AM), Roraima (RR), Rondônia (RO), Amapá (AP), Para (PA), Mato Grosso (MT), Tocantins (TO) and Maranhão (MA). This area has lower GDP per capita than the rest of the country, and is heavily dependent on agriculture and forestry.

Grain and livestock expansion in the North and Midwestern regions led to high rates of deforestation between 1995 and 2006 (Rivero *et al.*, 2009). Several recent studies highlight the relationship between agriculture and deforestation in Brazil, such as Reis and Guzmán (1992), Andersen *et al.* (2002), Diaz and Schwartzman (2005), Nepstad *et al.* (2007), Rivero *et al.* (2009), Araujo *et al.* (2009), Börner *et al.* (2010), Bowman *et al.* (2012), Assunção *et al.* (2013), Nepstad *et al.* (2014) and Filho *et al.* (2015).

Riveiro *et al.* (2009) and Margulis (2004) assert that livestock is the main driver of deforestation while Cardille *et al.* (2003) suggests that this result is not uniform across states, that livestock and crop production share the responsibility. Nepstad *et al.* (2001) and Quintanilha and Lee Ho (2005) also indicate timber activity as one of the main drivers. In a more recent analysis, Nepstad *et al.* (2014) indicate that interventions[11] and market restrictions on soybeans and livestock production led to a decrease in deforestation, although they are still important drivers.

Only a few papers highlight the relationship between deforestation and carbon dioxide (CO_2) emission using economic theory. Deforestation releases CO_2 and other greenhouse gases (GHGs) due to several factors such as tree burning, gradual decomposition of the forest biomass left on the ground and gradual release by commercialized forestry products while agricultural activity takes place (Aguiar *et al.*, 2012). FAO (2015) has endorsed the need for forest preservation since it is important not only for carbon sequestration but also for other ecological and environmental services. CO_2 concentration in the atmosphere is 18% higher

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[11] Interventions such as the Soy Moratorium (SoyM) in 2006, and the Cattle Agreement in 2010, constituted obstacles to deforestation despite the fact that neither are enforced, but instead are voluntary policies (Nesptad *et al.*, 2014; Gibbs *et al.*, 2014). The enforcement of new regulations such as the Brazilian Forest Code (FC), the Rural Environmental Registry of private properties (CAR), and surveillance by Brazilian Institute of the Environment and Renewable Natural Resources (IBAMA), have had positive impacts as deforestation control mechanisms (Gibbs *et al.*, 2014; Soares-Filho *et al.*, 2014; Hargrave and Kis-Katos, 2013).

than in 1980 and 27% higher than in 1960 (NOAA/ESRL, 2015), which suggests a rapid increase over the last 50 years.

Brazil participates in REDD+ (Reducing Emission from Deforestation and Forest Degradation), a clearing house for results-based payment agreements for reducing deforestation established by Parties to the United Nations Framework Convention on Climate Change (UNFCCC) with the objective of preserving the forest, ameliorating climate change and conserving biodiversity. Nepstad *et al.* (2007) focused on CO_2 emissions from land use in this region, aiming to evaluate the impact of REDD+[12] policies, and suggest a price of US$ 5.50 per ton of CO_2.

Nepstad *et al.* (2007) findings are based on a spatial dynamic modeling system proposed by Soares-Filho *et al.* (2006), named “SimAmazonia”, that integrates a soy rent, cattle rent and logging rent models embedded with biophysical characteristics, transportation cost and other factors. These rent models are loosely described on their paper but are explained in several other papers including Vera-Diaz *et al.* (2008) and Merry *et al.* (2009). To model soybean yields, Vera-Diaz *et al.* (2008) used an instrumental variable regression of soybean observed yields on simulated soybean yields, transportation cost, credit available to soybean agents (producers and trading companies), fertilizer expenditure, latitude and longitude. To investigate the Amazon timber industry, Merry *et al.* (2009) proposed a 30-year partial equilibrium simulation model that incorporates both economics and engineering aspects using data from the literature and several Brazilian institutions such as IBGE. Therefore, Nepstad *et al.* (2007) findings on CO_2 price are based mostly on simulations that include only soybean production, cattle production and timber sales, which underestimate agricultural activity in this region and therefore miss some of the agricultural benefits from deforested land. Soybean was responsible for only 34% of production value from crops in the Amazon Legal region in 2006, a slightly higher percentage (37%) was contributed by the cassava, corn, rice and sugarcane (IBGE, 2006), which are not considered in their model. Additionally, soybean production is clustered in the south region of the Amazon Forest, in the state of Mato Grosso, instead of uniformly spread across the Amazon Forest region.

Margulis (2004) described the interaction between deforestation and agricultural activities such as cattle and soybean production as dynamic, and provided an alternative estimate of the social cost of preventing deforestation at US$ 108 per hectare per year. The measure of benefits

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[12] May *et al.* (2011) and Heres *et al.* (2013) also discuss the role of REDD+ policies on Brazil.

from deforested land in this paper includes both direct use value (timber and non-timber products, and ecotourism), indirect use value (carbon stocking), option value (bioprospection) and existence value. These values are obtained from the literature rather than estimated by the author. At the average of 100 tons of C per hectare, this estimate is equivalent to $2.94 per ton, compared to the $5.50 of Nepstad.

Vera-Diaz and Schwartzman (2005), using budget information from the literature, constructed cash flows for soy, timber and cattle production and used Net Present Values (NPV) to reflect benefits from deforested land. Their estimate of the cost of a ton of CO_2 sequestrated ranged from US$0.80 to US$ 6.10. Börner, *et al.* (2010) used a similar method, but accounted for the dynamic land transition in the region when estimating the NPV. They conclude that even the most conservative carbon-offset price would lead to a decrease in deforestation due to the low per-hectare returns of the region.

A few papers have investigated Brazilian agriculture performance and its impact on Amazon Forest. Recently, Filho *et al.* (2015) showed that deforestation control will have small effects on the Brazilian food supply and that these effects could be neutralized by technological improvements. Basic rates of agricultural productivity gain in Brazil and in the Amazon region have been studied by Gomes and Braga (2008), Mendes *et al.* (2009), Bragagnolo *et al.* (2010), Gasques *et al.* (2012) and Rada and Valdes (2012)[13]. These papers did not include deforestation as an undesirable output.

We propose to estimate the trade-off between agriculture and forest preservation in the Amazon considering deforestation explicitly in the methodology as an undesirable output and accounting for all agricultural activities represented in the agricultural GDP. This will also allow estimation of a shadow price for CO_2 emissions from this source.

2.3.THE MODEL

Recent studies use directional distance functions to represent a technology that includes the joint production of an undesirable output, approximated both non-parametrically (Chung *et al.*, 1997; Macpherson *et al.*, 2010) and parametrically (Färe *et al.*, 2005; Färe *et al.*, 2006).

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[13] A few other studies such as Trindade *et al.* (2015), Fuglie (2010), estimated Brazil's TFP growth rates jointly with other countries.

Application of this framework to environmental issues such as the production of a byproduct like CO_2 and SO_2 has been fruitful[14]. Chung *et al.* (1997) raised questions about the inadequacy of long-established frameworks that do not consider undesirable outputs when measuring productivity, resulting in overstated productivity growth rates. Most of these papers have the intention of filling in the lack of information on price by estimating the shadow price of the undesirable output in terms of good outputs.

Alternative approaches and functional forms have been proposed to address production systems that include undesirable outputs. Färe *et al.* (2005) fit a stochastic quadratic directional distance function while Cuesta and Zofio (2005) and Cuesta *et al.* (2009) fit a translog distance function. A production technology useful for our purpose is described in Färe *et al.* (2005) and Macpherson *et al.* (2010). The production technology for agriculture uses inputs $\boldsymbol{x} \in \Re_{+}^{K}$ to produce outputs $\boldsymbol{u} \in \Re_{+}^{P}$. Some outputs are desirable, $\boldsymbol{y} \in \Re_{+}^{M}$ (agricultural GDP) and some are undesirable, $\boldsymbol{b} \in \Re_{\geq 0}^{R}$ (deforestation). The functional representation is

$$\vec{D}_o(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; g_y, g_b) = \max_{\lambda}\{\lambda: (\boldsymbol{y} + \lambda g_y, \boldsymbol{b} - \lambda g_b) \in P(\boldsymbol{x})\}, \qquad (1)$$

an output directional distance function for an output possibility set $P(\boldsymbol{x})$, where g_y and g_b are the directional vector $\boldsymbol{g} = (g_y, -g_b)$. The directional distance function[15] is non-negative in (***y***, ***b***), non-increasing and strongly disposable in $\boldsymbol{y}$, non-decreasing in $\boldsymbol{b}$, jointly weakly disposable and concave in (***y***, ***b***), and undesirable outputs are considered a byproduct of desirable outputs, known as the null-jointness hypothesis. The translation property, also referred to as homogeneity in outputs, is also satisfied if

$$\vec{D}_o(\boldsymbol{x}, \boldsymbol{y} + \lambda g_y, \boldsymbol{b} - \lambda g_b; g_y, -g_b) = \vec{D}_o(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; g_y, -g_b) - \lambda, \quad \lambda \in \Re \qquad (1i)$$

which states that increasing desirable outputs by λg_y while decreasing undesirable outputs by $-\lambda g_b$ is equivalent to subtracting the translation factor λ from the original directional distance

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[14] Badau *et al.* (2016) applies a framework similar to the one in this paper to a set of 141 countries aiming to find the shadow price of a ton of CO_2. Bokusheva *et al.* (2014) also estimated a distance function considering undesirable outputs.

[15] Appendix I, located at the end of the thesis describes these properties more extensively.

function. Equation (1) is represented in Figure 1 in output space considering a case with one desirable output y and one undesirable output b, and assuming a directional vector $\boldsymbol{g} = (g_y, -g_b) = (1, -1)$. The production possibility frontier represents all efficient units, captured by $\vec{D}_o(x, y, b; 1, -1) = 0$. For all observations in the output set $\vec{D}_o(x, y, b; 1, -1) \geq 0$; and the distance of each observation from the frontier is a measure of inefficiency.

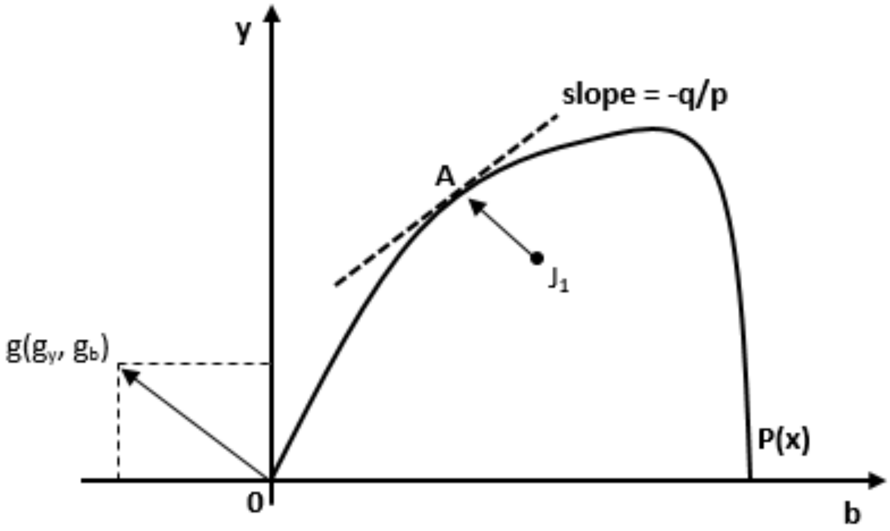

Figure 1 - Output Set - *P(x)*, and directional output distance function

Note: the scalars q and p represent price of undesirable and desirable outputs respectively.

The slope of a production possibility frontier is the marginal rate of transformation (MRT). Under profit maximization, producers would locate where the MRT is equal to the ratio of output prices (figure 1). For two desirables outputs the MRT is negative under strong disposability. When undesirable outputs are considered, weak disposability is assumed and the MRT is positive in the vicinity of preferred combinations (more desirable output and less undesirable output). This scenario, represented at point A in figure 1, is expected to occur when undesirable production has costly disposal fees or when restrictions are imposed on production of undesirable output.

The rate of transformation between desirable and undesirable output can be found by setting the total differential equal to zero and solving:

$$\nabla_{\boldsymbol{b}}\vec{D}_o \mathrm{d}\boldsymbol{b} + \nabla_{\boldsymbol{y}}\vec{D}_o \mathrm{d}\boldsymbol{y} + \nabla_{\boldsymbol{x}}\vec{D}_o \mathrm{d}\boldsymbol{x} = \mathrm{d}\vec{D}_o \tag{2i}$$

where on the frontier $\mathrm{d}\vec{D}_o = 0$ and $\mathrm{d}\boldsymbol{x} = 0$. The MRT between desirable output *m* and undesirable output *r* is

$$\frac{dy_m}{db_r} = -\frac{\partial\vec{D}_o/\partial b_r}{\partial\vec{D}_o/\partial y_m} \geq 0,\ \forall\, m\ and\ r. \tag{2ii}$$

Equation (2ii) is the opportunity cost of a change in b_r in terms of y_m. It is non-negative due to monotonicity in both desirable outputs ($\partial\vec{D}_o/\partial y_m \leq 0$) and undesirable output ($\partial\vec{D}_o/\partial b_r \geq 0$).

Let $\boldsymbol{p} = (p_1, p_2, \dots, p_M) \in \Re_+^M$ and $\boldsymbol{q} = (q_1, q_2, \dots, q_R) \in \Re_+^R$ represent the prices of desirable and undesirable outputs. The dual relationship between the revenue function and the output distance function is (Färe *et al.* (2006))

$$R(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{q}) = max_{\mathbf{y},\mathbf{b}}\{\boldsymbol{py} - \boldsymbol{qb} : \vec{D}_O(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \geq 0\} \tag{3}$$

or

$$\overrightarrow{D_O}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; g) = min_{p,q}\left\{\frac{R(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{q}) - (\boldsymbol{py} - \boldsymbol{qb})}{\boldsymbol{p}g_y + \boldsymbol{q}g_b}\right\} \tag{4}$$

Note that in the presence of undesirable outputs the revenue function could be negative. Using the envelope theorem,

$$\begin{aligned} (\boldsymbol{p}g_y + \boldsymbol{q}g_b)\nabla_{\boldsymbol{b}}\vec{D}_O(x, y, b, g) &= \boldsymbol{q} \geq 0 \\ (\boldsymbol{p}g_y + \boldsymbol{q}g_b)\nabla_{\boldsymbol{y}}\vec{D}_O(x, y, b, g) &= -\boldsymbol{p} \leq 0 \end{aligned} \tag{5}$$

then the shadow price of an undesirable output, q_r, in terms of a desirable output y_m is

$$q_r = -p_m \left[\frac{\partial \vec{D}_O(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}, \boldsymbol{g})/\partial b_r}{\partial \vec{D}_O(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}, \boldsymbol{g})/\partial y_m} \right] \qquad (6)$$

where p_m is the known market price of the desirable output y_m, $m = 1, 2, \ldots, \mathrm{M}$ and $r = 1, 2, \ldots, \mathrm{R}$. Equation (6) is the shadow price or the opportunity cost of decreasing production of undesirable outputs, evaluated at market prices of a desirable output. Expression (6) is non-negative given that the marginal rate of transformation in brackets is negative due to the monotonicity property of the distance function with respect to the desirable and undesirable outputs. Given estimates of the marginal rates of transformation, we use this equation to infer the shadow prices of deforestation, and from that the estimates of the cost of CO_2 released by deforestation.

2.4.THE APPLICATION

We use the Agricultural Census of 2006, made available by IBGE, which includes 590[16] municipalities in nine states; Amapá (AP), Acre (AC), Amazonas (AM), Mato Grosso (MT), Maranhão (MA), Tocantins (TO), Pará (PA), Rondônia (RO), and Roraima (RR). Desirable outputs and inputs were obtained from IBGE while deforestation was obtained from the National Institute for Space Research (INPE). Descriptive statistics are in Table 1. Appendix 2.A presents more information on these variables.

Deforestation is measured in hectares. Margulis (2004) suggests that deforestation might occur over three years, and be detected only in the third year of the process, depending on the process of deforestation used. It is possible that agricultural activities would be occurring during this process with revenue from both agriculture and timber sales during this period. This leads us to measure deforestation in 2006 as the average of the previous three years (2004-2006). The states of Roraima (RO), Mato Grosso (MT), Pará (PA) and Maranhão (MA) have higher absolute and per hectare measures of deforestation. Over three years, MT and PA each contributed more than 30% of the total deforested area, jointly approximating 69% of what was deforested in this period.

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[16] IBGE states that the Legal Amazon includes 771 municipalities, however some municipals did not have any forest in 2006.

Table 1 - Descriptive Statistics for Agricultural Production, Inputs and Deforestation in 590 Municipalities in the Legal Amazon, Brazil, 2006.

Variable	Description	Mean	Std. Dev.	Min	Max
b_1	Av. Deforestation 2004-06 (ha)	3438.90	7917.37	0.00	108406.70
y_1	Ag. GDP (US$ 1000)	8862.21	11224.85	410.47	119599.20
x_1	Capital (units)	1637.64	1374.53	49.00	11283.00
x_2	Labor (units)	3870.83	3776.78	126.00	39883.00
x_3	Irrigation (ha)	415.08	1381.82	0.00	12850
x_4	Credit (US$ 1000)	2449.30	12485.24	0.00	263341
z_1	Shared of family owned farms	0.86	0.13	0.03	1.00
z_2	Total forest area in 2006 (km2)	5190.67	13459.67	0.10	144093.00
z_3	Total hydrology area in 2005 (km2)	184.07	480.06	0.00	4499.90

Source: Deforestation is from PRODES/INPE, Ag. GDP is from IPEA, and grains, timber, sold heads, capital and labor are from the Agricultural Census (IBGE).

Municipalities in the state of Mato Grosso had larger agricultural production in 2006, with an average agricultural GDP of $15,992 thousand dollars. This state was responsible for 39% of what was produced in the 590 municipalities considered in this research, as reflected in its agricultural GDP. Table A1 and Figure A1 display the distributions of agricultural GDP and average deforestation during the three years from 2004 to 2006.

We were able to obtain information on four inputs; labor as the number of employees; area of land irrigated (in hectares), total credit obtained by the municipality (in US$ 1000) and capital obtained by summing the number of pieces of equipment and number of landowners in the municipality, following Bragagnolo *et al.* (2010). Following Färe *et al.* (2005), all variables are normalized by their means[17], aiming to achieve convergence in the stochastic estimation.

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[17] For a hypothetical municipality that uses mean inputs and produces mean outputs, the input and output variables would be $(x, y, b) = (1,1,-1)$. It implies that Figure 1 is in normalized values. Thus observation J1 (illustrated in Figure 1) can be expanded by $\lambda^* \bar{y}$, and contracted by $\lambda^* \bar{b}$ simultaneously.

2.4.1. Empirical Strategy

We approximate the distance function (Eq. 1) using a quadratic functional form, with the subscript $i = (1,2,\dots,N)$ representing municipalities

$$\vec{D}_{o_i}(x_i, y_i, b_i; 1, -1) = \alpha_0 + \sum_{k=1}^{4} \alpha_k x_{ki} + \frac{1}{2}\sum_{k=1}^{4}\sum_{l=1}^{4} \alpha_{kl} x_{ki} x_{li} + \beta_1 y_{1i}$$
$$+ \frac{1}{2}\beta_{11} y_{1i} y_{1i} + \theta_1 b_{1i} + \frac{1}{2}\theta_{11} b_{1j} b_{1i} + \mu_{11} y_{1i} b_{1i} + \sum_{k=1}^{4} \delta_{km} x_{ki} y_{1i} + \sum_{k=1}^{4} \varphi_{kr} x_{ki} b_{1i} \tag{7}$$

where x_{ki} are labor, capital, irrigation and credit, y_{1i} is agricultural GDP, b_{1i} is deforestation, and α's, β's, θ's, δ's, φ's and μ's are parameters to be estimated. As Färe *et al.* (2005) we use the directional vector $\boldsymbol{g} = (g_y, -g_b) = (1, -1)$ representing a simultaneous expansion in desirable outputs and a contraction of undesirable outputs. The symmetry and translation property in outputs are imposed before estimation, requiring the following restrictions

$$\beta_1 - \theta_1 = -1, \qquad \beta_{11} - \mu_{11} = 0, \qquad \theta_{11} - \mu_{11} = 0, \qquad \delta_{k1} - \varphi_{k1} = 0 \tag{7i}$$

where *k =1, ...,4* are inputs. We estimated equation (7) after imposing (7i) as

$$-\lambda_i = \vec{D}_{o_i}(x_i, y_i + \lambda_i, b_i - \lambda_i; 1, -1) + \epsilon_i, \tag{8}$$

where λ_i is the translation factor; in our case, $\lambda_i = b_{1i}$[18]. The following quadratic functional form with symmetry and translation properties imposed is estimated

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[18] Summary and Weber (2012) and Weber and Xia (2011) have considered as the translation factor the negative of a desirable output (i.e. $\lambda_i = -y_{1i}$), since they only have desirable outputs. Kumar and Managi (2010) include undesirable outputs in their analysis and have used as the translation factor a desirable output (i.e. $\lambda_i = y_{1i}$). These specifications would change equation (7i).

$$
\begin{aligned}
-b_{1i} = \alpha_0 &+ \sum_{k=1}^{4} \alpha_k x_{ki} + \frac{1}{2}\sum_{k=1}^{4}\sum_{l=1}^{4} \alpha_{kl} x_{ki} x_{li} + \beta_1 y'_{1i} + \frac{1}{2}\beta_{11} y'_{1i} y'_{1i} + \theta_1 b'_{1i} \\
&+ \frac{1}{2}\theta_{11} b'_{1i} b'_{1i} + \sum_{k=1}^{4} \delta_{km} x_{ki} y'_{1i} + \sum_{k=1}^{4} \varphi_{k1} x_{ki} b'_{1i} + \epsilon_i
\end{aligned}
\tag{9}
$$

where the prime (') represents the normalization with respect to λ_i that captures the translation property, $y'_{mi} = (y_{mi} + \lambda_i)$ and $b'_{ri} = (b_{ri} - \lambda_i)$ and $\lambda_i = b_{1i}$. Desirable output y'_{1i} is agricultural *GDP*, inputs x_{ki} are capital, labor, irrigation and credit and $b'_{1\mathrm{i}}$ is deforestation. We use the translation property to recover parameters of the output used to impose the translation property. To estimate the parameters of equation (9) we use corrected ordinary least squares (COLS) and maximum likelihood (MLE) approaches.

Using equation (6) and given prices for the desirable output, p_m, the shadow price for an undesirable output in each municipality i is[19]:

$$
q_{b_{1i},y_{1i}} = -p_{1i}\left[\frac{\theta_1 + \theta_{11} b_{1i} + \sum_{k=1}^{4} \varphi_{k1} x_{ki} + \mu_{11} y_{1i}}{\beta_m + \beta_{11} y_{1i} + \sum_{k=1}^{4} \delta_{km} x_{ki} + \mu_{11} b_{1i}}\right]. \tag{10}
$$

where there are $k = 1, \ldots, 4$ inputs. Equation (10) is used in this study to estimate the shadow price of reducing one hectare of deforestation in terms of agricultural GDP foregone.

Equation (9), was estimated using Corrected Ordinary Least Squares (OLS) and Maximum Likelihood Estimation (MLE)[20]. Estimates of the COLS were used as starting values for the MLE procedure. In the first approach, inefficiency is calculated by modifying the error term. This error term, appended to equation (9), is $\epsilon_i = -u_i + v_i$, where $= -u_i$ represents an inefficiency term, and v_i, is the random error. Since $E(u_i) \neq 0$, the estimate of the intercept, α_0, is biased although those of the slope coefficients, $\tilde{\beta}$, are not. The intercept is adjusted upwards, post estimation, by the $\min\{\widehat{\epsilon_i}\}$, so that the adjusted function bounds observations from above. On the other hand, MLE does not incur on parameter biases given that we assume

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[19] To express this in units of output we need to multiply by the ratio of means, as the outputs had been normalized by their means.

[20] Appendix II, located at the end of the thesis, describes these methods more extensively in addition to guide you to the references on these methodologies.

a half-normal distribution ($u_i \sim N^+(0, \sigma_{ui}^2)$) for the inefficiency term, as described in Kumbhakar et. (2015). The distribution of the efficiency error, specifically σ_{ui}^2, is assumed to be a function of exogenous variables (z_i). Therefore, the coefficients estimated for z_i are not to be interpreted as a marginal effect on $E(u_i)$, although their signs give the direction of their impact on $E(u_i)$. In this paper, we represent z_i by forest area in 2006, hydrological area in 2006, and share of farms that are family owned. The estimation was done using Stata 14 following the command *sfmodel* suggested by Kumbhakar et. (2015) and *sfcross* suggested by Belotti *et al.* (2012). Estimates are in Table 2.

2.5.RESULTS AND DISCUSSION

We estimated the quadratic specification for the directional distance function in equation (9)[21] using COLS and MLE. Parameters estimated are displayed in Table 2.

Table 2 – COLS and MLE Parameter Estimates for Directional Distance Function in Municipalities of the Legal Amazon, Brazil in 2006

Coefficient	*Variable*	*COLS*	*MLE*
$\boldsymbol{\beta_1}$	y_1	-0.8388***	-0.8315***
		(0.0176)	(0.0199)
$\boldsymbol{\beta_{11} = \theta_{11} = \mu_{11}}$	y_1^2	-0.0150***	-0.0147***
		(0.0034)	(0.0033)
$\boldsymbol{\alpha_2}$	x_2	-0.2158**	-0.1816*
		(0.1108)	(0.1091)
$\boldsymbol{\alpha_{22}}$	x_2^2	-0.1009	-0.097
		(0.1000)	(0.0970)
$\boldsymbol{\alpha_1}$	x_1	0.4901***	0.4921***
		(0.1589)	(0.1540)
$\boldsymbol{\alpha_{11}}$	x_1^2	-0.1737	-0.1583
		(0.2027)	(0.1967)

---

[21] As robustness check, we estimate the same model using a different econometric approach, Generalized Method of Moments, and using MLE considering different ways of building the variable capital. Summary and Weber (2012) and Weber and Xia (2011) suggest possible existence of endogeneity. These papers used datasets that have both temporal and cross-section units, which allows use of the procedure in Atkinson *et al.* (2003) to recover inefficiency. Even though efficiency estimation is not the main concern of this paper we use this method to allow for possible endogeneity of outputs and inputs. We also use the Hansen J statistic to test overidentification of our instruments. We learn that MLE results are robust to these specifications given the median and average shadow prices are similar among models. Therefore, we prefer the MLE model, especially because it allows us to incorporate additional heterogeneity through the efficiency component of the error term.

$\boldsymbol{\alpha_{12}}$	x_1x_2	0.1369	0.1226*
		(0.1359)	(0.1320)
$\boldsymbol{\delta_{21} = \varphi_{21}}$	y_1x_2	0.0265	0.0353
		(0.0204)	(0.0213)
$\boldsymbol{\delta_{11} = \varphi_{11}}$	y_1x_1	-0.0021**	-0.0143***
		(0.0277)	(0.0288)
$\boldsymbol{\alpha_4}$	x_4	0.0756***	0.0689***
		(0.0310)	(0.0298)
$\boldsymbol{\alpha_{44}}$	x_4^2	-0.0071***	-0.0072***
		(0.0004)	(0.0004)
$\boldsymbol{\alpha_{34}}$	x_3x_4	-0.0089***	-0.009***
		(0.0020)	(0.0018)
$\boldsymbol{\delta_{41} = \varphi_{41}}$	y_1x_4	0.0361	0.0365
		(0.0037)	(0.0036)
$\boldsymbol{\alpha_{14}}$	x_1x_4	0.0060	0.0071
		(0.0205)	(0.0199)
$\boldsymbol{\alpha_{24}}$	x_2x_4	-0.0163***	-0.0142***
		(0.0240)	(0.0237)
$\boldsymbol{\alpha_3}$	x_3	0.1555***	0.1478***
		(0.0252)	(0.0241)
$\boldsymbol{\alpha_{33}}$	x_3^2	-0.0128	-0.0125***
		(0.0232)	(0.0019)
$\boldsymbol{\alpha_{23}}$	x_2x_3	-0.0135	-0.0126
		(0.0239)	(0.0223)
$\boldsymbol{\alpha_{13}}$	x_1x_3	-0.0143	-0.0140
		(0.0174)	(0.0166)
$\boldsymbol{\delta_{31} = \varphi_{31}}$	y_1x_3	0.0233***	0.0238***
		(0.0041)	(0.0039)
$\boldsymbol{\alpha_0}$	Constant	0.0996	0.1500**
		(0.0646)	(0.0678)
$\boldsymbol{\sigma_u}$			
$\boldsymbol{\vartheta_1}$	z_1	-	15.2584*
		-	(8.4439)
$\boldsymbol{\vartheta_2}$	z_2	-	7.25E-05***
		-	(2.79E-05)
$\boldsymbol{\vartheta_3}$	z_3	-	-0.0036*
		-	(0.0020)
$\boldsymbol{\vartheta_4}$	Constant	-	-18.0230**
		-	(8.0975)
$\boldsymbol{\sigma_v}$		-	0.4669***
		-	(0.0140)
N		590	590

Note: COLS parameters used as starting values for MLE. Standard error in parenthesis; *** for p-value smaller than 0.01, ** smaller than 0.05, and * smaller than 0.1. The dependent variable is the negative of average deforestation. All models include eight state dummies.

COLS estimation has eleven statistically significant parameters out of 21 (excluding state dummies), while MLE has thirteen. The properties of monotonicity and null-jointness[22] were checked at each data point after estimation with the MLE estimates presenting fewer violations. A Likelihood Ratio test of 15.37 indicates that MLE estimates with a half-normal distribution for the one-sided error term are superior to the COLS estimates at 1% of significance (critical value is 14.32). The analysis that follows uses the MLE estimates. Only six observations did not satisfy monotonicity on desirable output for both MLE and COLS while four observations do not satisfy monotonicity on undesirable output for the COLS and three for the MLE.

An estimate of the distance of each municipality from the frontier is obtained from equation (9), and is interpreted as a measure of inefficiency. The average distance estimated for the region was 0.087. This means that, on average, agricultural GDP could be expanded by 8.7% (an average of US$ 771 thousand) while simultaneously decreasing deforestation by 8.7% (an average of 299.12 hectares). A higher share of family owned farms and of forested area is associated with higher inefficiency while hydrological area with lower. Resources for one municipality are not in fact identical to those of the others, so the inefficiency estimates are at least in part an indicator of the heterogeneity across these municipalities not captured by the variables included in our model.

Using equation (10) and the estimated parameters we calculate the shadow price at each observation. The results are displayed in Figures 2 and 3 and in Tables 3 and 4.

---

[22] Null-jointness property was not imposed but checked after. This property was violated in less than 32% of the sample. Both COLS and MLE have satisfied the concavity property.

Table 3 –Average Estimates of Shadow Prices for Deforestation and CO_2, Legal Amazon and 'arc of deforestation', Brazil, 2006.

	COLS[a]	**MLE**[a]
$Median\ -\boldsymbol{p_1}\left[\frac{\partial D_O(x,y,b,g)/\partial b_1}{\partial D_O(x,y,b,g)/\partial y_1}\right]$ *in* US\$	556.82	577.80
$Mean\ -\boldsymbol{p_1}\left[\frac{\partial D_O(x,y,b,g)/\partial b_1}{\partial D_O(x,y,b,g)/\partial y_1}\right]$ *in* US\$	770.72	796.81
$at\ -\boldsymbol{p_1}\left[\frac{\partial D_O(1,1,1,g)/\partial b_1}{\partial D_O(1,1,1,g)/\partial y_1}\right]$ *in* US\$ [b]	705.73	728.84
CO_2 Shadow price in US\$ *(base on median, r =0.1)*	15.17	15.74
CO_2 Shadow price in US\$ *(base on median, r =0.06)*	25.29	26.24

Note: [a] Shadow prices are calculated only using observations that satisfied monotonicity; shadow prices would be slightly smaller if calculated with all observations as there are only six violations for GDP and 3 (4 in COLS) for deforestation in the MLE estimation. COLS parameters used as starting values for MLE. [b] The shadow price evaluated at the mean of all observations, $(x, y, b) = (1,1,1)$.

On average, the MLE estimates indicate that US\$ 796.81[23] in agricultural GDP has to be given up each year to preserve a hectare of forest (*i.e.,* to decrease deforestation by one hectare). The frequency distribution of these estimates shows a concentration of municipalities with shadow price around this average, as displayed in Figures 2 and 3.

---

[23] In 2006 the revenue given up to preserve one forested hectare is \$796.81. To keep it preserved this revenue has to be given up every year. This value was obtained considering only the observations that satisfied monotonicity in both desirable and undesirable outputs. An average shadow price of US\$ 580.20 is found when considering all observations.

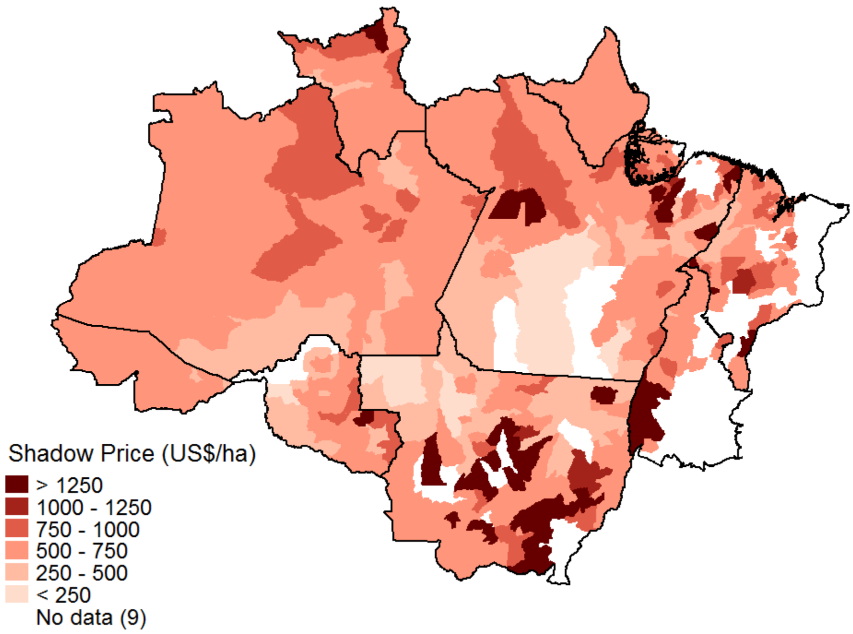

Figure 2 – Estimated shadow prices of deforestation in terms agricultural GDP in the Legal Amazon, Brazil, 2006 (MLE)

Table 3 illustrates that 50% of the sample has a shadow price equal or smaller than US$ 557.80. About 86% of the area that was deforested appears to have an opportunity cost of less than $796. The cost to preserve the forest in perpetuity, using a discount rate of 10% and the median shadow price estimated, is the present value of that future revenue stream, or US$5,578 per hectare. Using a more restricted dataset and a simpler methodology based on budget data and calculating Net Present Value, Margulis (2004) estimates a much lower annual social cost of approximately US$108 (US$135 in 2006 currency) per hectare preserved.

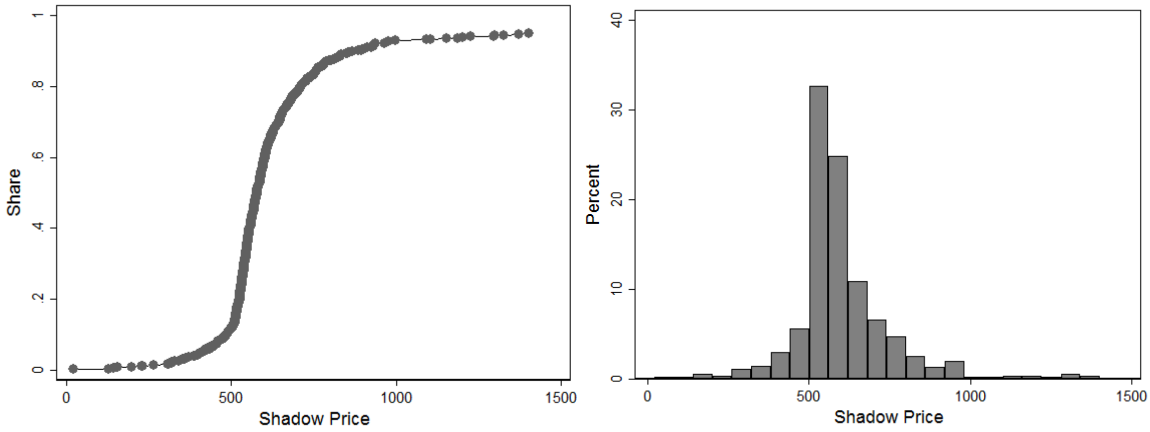

Figure 3 – Cumulative distribution and histogram of estimated shadow prices of forest preservation in terms of agricultural GDP for the Legal Amazon, Brazil, 2006

Note: The higher 5% percentile is not included in these graphs for scale reasons.

Table 4 suggests that municipalities in Mato Grosso (MT), Roraima (RR), Maranhão (MA) and Para (PA) have higher shadow prices than municipalities in other states. These municipalities are in the lower geographical boundary of "arc of deforestation" region, where agriculture has been fast developing during the last decades. As expected, then, municipalities with higher agricultural production rates have larger tradeoffs (shadow prices).

Table 4 – Average Shadow Prices of Forest Preservation in Terms of Agricultural GDP (US$), in the Legal Amazon, Brazil, 2006

	$ShadowPrice_{def,gdp}$ (US$)			
	Mean	***Standard Deviation***	***Minimum***	***Maximum***
Acre (AC)	552.87	57.00	435.65	690.33
Amazonas (AM)	603.16	117.23	309.39	931.67
Amapá (AP)	554.89	57.95	504.82	710.92
Maranhão (MA)	744.19	904.82	318.21	9950.55
Mato Grosso (MT)	1252.85	2311.84	126.72	17064.06
Para (PA)	669.38	340.87	21.98	2748.87
Rondônia (RO)	616.02	264.04	197.00	2212.24
Roraima (RR)	974.83	1420.30	489.71	6088.08
Tocantins (TO)	689.47	653.52	491.56	5664.03
Legal Amazon (LA)	**796.81**	**1206.76**	**21.98**	**17064.06**

2.5.1. A shadow price of Carbon Dioxide (CO2)

We use the estimates above to approximate the shadow price of CO_2 emissions from expanding agricultural activities in the Legal Amazon. This is very relevant information if global markets for CO_2 were developed. We then compare our results with others in the literature (Vera-Diaz and Schwartzman (2005), Nepstad *et al.* (2007), Börner *et al.* (2010) and Assunção *et al.* (2013)) and with the CO_2 price estimates used by the Brazilian Ministry of the Environment[24].

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[24] Technical note N°. 40, Departamento de Politicas para o Combate ao Desmatamento, Ministerio do Meio Ambiente (MMA/DPCD), 2012. In this document, Brazilian government discuss how much should have raised for the "Fundo Amazonia" to preserve the forest. The estimations are based on data from PRODES/INPE and, for 2012 considering US$ 5 per ton of CO_2, US$ 2.5 billion has to be raised to decrease deforestation to zero.

The Brazilian government estimates that on average, one hectare of Brazilian forest sequesters 100 tons of carbon (Ministerio do Meio Ambiente, 2012; Amazon Fund, 2015). One ton of carbon is equivalent to 3.67 tons of CO_2, therefore one average hectare of Brazilian forest sequesters 367 tons of CO_2. Using this carbon intensity coefficient, our results imply an estimated shadow price of US$1.57 per ton of CO_2 sequestered per year in a median hectare ($2.16 for the average hectare). This means that to retain (rather than release by deforestation) one ton of CO_2 indefinitely on an average hectare, US$2.16 of agricultural GDP has to be foregone every year. Assuming a social discount rate in the range of 5% to 10%, as in Margulis (2004), Vera-Diaz and Schwartzman (2005), Nepstad *et al.* (2007), and Börner *et al.* (2010), this implies an average shadow price of CO_2 ranging from US$ 21.7 to US$ 43.2 per ton.

Given our distribution of estimates 86% of the municipalities have annual tradeoffs lower than the average of US$2.16 per ton of CO_2 per year. These municipalities represent 88% of the average deforestation that occurred during 2004-2006. These estimates are much higher than the shadow price of US$ 5.00 per ton of CO_2 used by the Amazon Fund to raise funds to preserve the forest. At a 10% discount rate and a carbon content of 155 tons of Carbon per hectare as in Vera-Diaz and Schwartzman (2005), the price of a ton of CO_2 goes down to US$ 14.00 in perpetuity.

2.6.CONCLUSIONS

The benefits of preserving a hectare of Amazon forest are not restricted to Brazilians, but accrue to mankind as a whole. However, the opportunity costs of preservation fall upon the Brazilians themselves, who must trade off potential agricultural income forgone in exchange for a hectare of forest preserved. This paper evaluates the tradeoff between forest preservation and agricultural expansion in the Legal Amazon region of Brazil, by estimating the aggregate municipality-level technology using directional distance functions. Data is for more than 500 municipalities obtained from the Agricultural Census of 2006 and from the National Institute for Space Research (INPE/PRODES) for deforestation. The directional distance function was approximated by a flexible quadratic form and estimated using stochastic methods.

The tradeoff estimates differ widely across municipalities, reflecting the diversity of resources and agricultural activities. On average, we estimate that US$796.81 in agricultural GDP has to be foregone annually to preserve one additional hectare of forest. This translates to an average shadow price of US$21.70 per ton of CO_2 sequestered in perpetuity (at a social discount rate of 0.10 and 100 tons per hectare of carbon sequestered). This average is much higher than the US$5 used in official transactions. It is important to notice that that the distribution of estimated shadow prices is skewed and that almost 90% of the estimated shadows are lower than the average. These values depend on the assumptions made on carbon content per hectare, discount rate and exchange rates.

Although our estimates of the opportunity cost embody all agricultural activities contributing to agricultural GDP rather than only livestock, timber and grains production as in most of the literature, we do not include forest benefits such as existence value and benefits from preservation of habitat and biodiversity. Thus, our estimates should be interpreted as a lower-bound opportunity cost.

CHAPTER 3

THE EFFECT OF TECHNICAL CHANGE ON THE TRADEOFF BETWEEN AGRICULTURE AND THE AMAZON FOREST IN THE BRAZILIAN "ARC OF DEFORESTATION"

3.1.INTRODUCTION

Brazil encompasses the largest tropical forest in the world, the Amazon forest – corresponding to 13% of the world's forest area and around 60% of Brazil's surface. Strong agricultural expansion, starting in the 1990s, has impacted this area greatly. In the literature, grains, livestock and timber production are indicated as the main drivers of deforestation in the agricultural frontier in this region, also known as the "arc of deforestation" (Riveiro *et al.* (2009); Margulis (2004); and Nepstad *et al.* (2001)).

However, this region, as well as Brazil overall, has been experiencing high rates of technical change. Bragagnolo *et al.* (2010) reports an average technical progress for the states that includes the municipalities we are considering as the "arc of deforestation" of around 7.2% a year. A few studies have investigated technical change effects on deforestation. Filho *et al.* (2015) showed that deforestation control will have small effects on the Brazilian food supply and that these effects could be neutralized by technological improvements.

Villoria *et al.* (2014) highlights the role of technical change (and productivity change) on forest preservation and the lack of empirical work to evaluate it. In this paper, we estimate the rate of technical change seeking to shed light on its effect on the tradeoff between agricultural production and forest preservation in the "arc of deforestation" in the Brazilian Amazon. We are also interested on whether technical change in agriculture has been biased toward agricultural production or deforestation.

To evaluate the effects of technical change on the tradeoff we first estimate the production possibility frontier (PPF) for agricultural production and forest preservation at municipality-level for the period from 2003 to 2015. Then, we identify whether technical change was progressive or regressive by analyzing the marginal change on the PPF, as proposed by Färe and Karagiannis (2014). Finally, we propose a primal-oriented technical change bias definition based on Fulginiti (2010) to evaluate whether technical change was bias against deforestation or not. This paper contributes to the literature by shedding light on the influence of technical change on forest preservation.

Our results suggest that these municipalities have been experiencing progressive technical change in agriculture of, on average, around 4.6% a year. It means that, on average, due to innovations agricultural outputs expanded 4.6% while deforestation contracted by 4.6%. Technical change has also led to a higher marginal rate of transformation between agricultural outputs and deforestation. In other words, technical change has been biased toward agricultural outputs and against deforestation.

3.2.BACKGROUND

In the literature, the "arc of deforestation" is defined as the municipalities in the agricultural frontier in the northern region of Brazil with high level of deforestation. In this paper, we investigate technical change in agriculture when deforestation is also considered. We use information of two hundred municipalities in seven states that have high level of deforestation[25]: Acre (AC), Amazonas (AM), Pará (PA), Rondônia (RO), Mato Grosso (MT), Maranhão (MA) and Roraima (RR). Figure 1 displays total deforestation during the period from 2001 to 2015 at municipality scale. To better understand the magnitude of the deforestation in this region, in 2015, 1675.23 square miles were logged in the "arc of deforestation", it represents 2.2% of the state of Nebraska (77,421 square miles).

[2525] In the application section we describe how we define this sample.

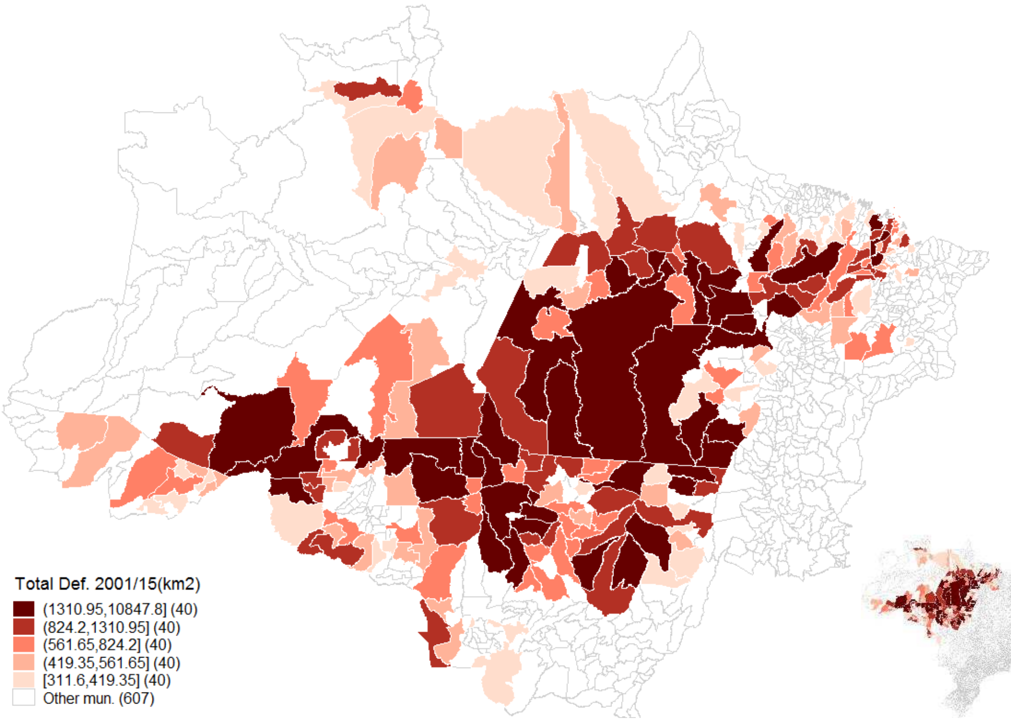

Figure 1 – Sum of deforestation (in km^2) for the two-hundred municipalities in the "arc of deforestation" in the northern region of Brazil.

Note: "Other mun." in these maps represent municipalities that are not being using on the estimation of Equation (1). In the application section we describe how we defined the two-hundred municipalities.
Source: Own elaboration using Stata 14.

Rivero *et al.*, (2009) assert that high rates of deforestation between 1995 and 2006 were caused partially by grain and livestock expansion in the North and Midwestern regions. In addition to these two activities, timber revenue has also been indicated as a motivation to increase deforestation [Riveiro *et al.* (2009); Margulis (2004); Cardille *et al.* (2003); Nepstad *et al.* (2001); and Quintanilha and Lee Ho (2005)]. Other studies also highlight this relationship, between agriculture and deforestation in Brazil, such as Reis and Guzmán (1992), Andersen *et al.* (2002), Diaz and Schwartzman (2005), Nepstad *et al.* (2007), Rivero *et al.* (2009), Araujo *et al.* (2009), Börner *et al.* (2010), Bowman *et al.* (2012), Assunção *et al.* (2013), Nepstad *et al.* (2014) and Filho *et al.* (2015).

On the role of technical change in forest preservation Villoria *et al.* (2014) presents a literature review and a discussion of studies that investigate this topic. They suggest that technical change (and productivity change) could lead to two opposite effects on forest preservation; higher level of deforestation as commercial activity is expanded, or lower deforestation due to a less land intensive production (input substitution). They argue that to conclude whether it is the first or the second hypothesis that takes place in a particular region empirical work has to be done.

Filho *et al.* (2015) investigate whether Brazil can increase food supply without increasing deforestation. They highlight that conversion of low-yield pasture area and a low exogenous technical change can neutralize the effect of controlling deforestation on agricultural supply. To obtain these results, they have used a Computable General Equilibrium (CGE) model of Brazil to model land use over 20 years. One of drawbacks of these type of models is the rigidity on the assumptions made about the agricultural technology, input and output substitution and its relationship with forestry area – constant elasticity of substitution (CES).

On productivity change, several papers have studied Brazilian agriculture. Bragagnolo *et al.* (2010) estimate Total Factor Productivity (TFP) for Brazilian agriculture using a panel of municipalities and agricultural census data (1975, 1985, 1995 and 2006). They fit a translog production function to obtain the TFP and its several components including technical change. For Brazil, they found an average technical progress of around four percent. Using their state averages, we calculated the average technical progress of the states that include municipalities included here as the “arc of deforestation” of around seven percent a year. State average of technical progress for this sample ranges from 3.9% (Maranhão) to 10.2% (Roraima) a year.

On a broader context, Gasques and Conceicao (1997), Gasques *et al.* (2004), Gasques *et al.* (2009) and Fuglie (2010) have found TFP rates higher than three percent for Brazil. Gasques *et al.* (2014) argue that a favorable international scenario, public research, and credit availability had an important role in these results. Rada and Valdes (2012) has also found positive TFP mainly driven by technical change (around 4%) for the last decades. Mendes *et al.* (2009) and Trindade and Fulginiti (2015) have found a lower TFP, 1% growth rate for 1985-2004 and 2% growth rate for 1969-2009 respectively. None of these studies have estimated technical change at municipality level or have considered the effects of agriculture on the environment.

Additionally, Gomes and Braga (2008) have investigated which factors explained the Total Factor Productivity of the Legal Amazon region at a state level data. They have found that infrastructure and credit made available by a regional institution (Fundo Constitutional de Financiamentos do Norte) has contributed to TFP, leading to higher rates.

The harmful effects from production of *goods* on the environment has been addressed using directional output distance functions where undesirable outputs (i.e. pollution) and desirable outputs (i.e. *goods* production) are being treated asymmetrically. Chung *et al.* (1997) argue that productivity change rates estimated using conventional methods without considering the harmful effects on the environment might be overestimated. Only a few studies have included undesirable outputs to evaluate productivity change in agriculture such as Färe *et al.* (2006) and Kabata (2011) for United States and Flavigna *et al.* (2013) for Italy. In this paper, we estimate the rate of technical change in a set of municipalities in the northern agricultural frontier of Brazil seeking to identify whether bias has been toward agricultural production or towards deforestation.

3.3.THE MODEL

Several studies have used directional distance functions to represent a technology that includes the joint production of an undesirable output (Chung *et al.*, 1997; Macpherson *et al.*, 2010; Färe *et al.*, 2005; Färe *et al.*, 2006). Chung *et al.* (1997) raised questions about the inadequacy of long-established frameworks that do not consider undesirable outputs when measuring productivity, resulting in overstated productivity growth rates. We will compare technical change estimated in this paper with estimates reported in the literature. The production technology that involves both desirable and undesirable outputs is describe in Färe *et al.* (2005) and will be summarized here[26].

The agricultural production technology uses inputs $x_i = (x_{1i}, \dots, x_{Ki}) \in \Re_+^K$ to develop outputs $u_i = (u_{1i}, \dots, u_{Vi}) \in \Re_+^P$. Some outputs are desirable $y_i = (y_{1i}, \dots, y_{Mi}) \in \Re_+^M$, such as grains production and some undesirable $b_i = (b_{1i}, \dots,\ b_{Ri}) \in \Re_+^R$, such as deforestation. The subscript $i = (1,2, \dots, N)$ represents the observed unit. The subscript *t* for time is dropped in this sections for simplicity. The mathematical representation of the directional output distance function is given by

$$\vec{D}_o(x, y, b, t; 1, -1) = max\{\alpha\colon (y + \alpha g_y, b - \alpha g_b)\ \epsilon\ P(x)\} \qquad (1)$$

an output directional distance function for an output possibility set $P(\boldsymbol{x})$, where g_y and g_b are the directional vector $\boldsymbol{g} = (g_y, -g_b)$. The directional distance function is non-negative in (***y*, *b***), non-increasing and strongly disposable in ***y***, non-decreasing in ***b***, jointly weakly disposable

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[26] Appendix I, located at the end of the thesis describes these properties more extensively.

and concave in (***y*, *b***), and undesirable outputs are considered a byproduct of desirable outputs, known as the null-jointness hypothesis. The translation property, also referred to as homogeneity in outputs, is also satisfied if

$$\vec{D}_o(\boldsymbol{x}, \boldsymbol{y} + \alpha g_y, \boldsymbol{b} - \alpha g_b; g_y, -g_b) = \vec{D}_o(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; g_y, -g_b) - \alpha, \quad \alpha \in \Re \tag{1i}$$

which states that increasing desirable outputs by αg_y while decreasing undesirable outputs by $-\alpha g_b$ is equivalent to subtracting the translation factor α from the original directional distance function. Equation (1) is represented in Figure 2 in output space considering a case with one desirable output *y* and one undesirable output *b*, and assuming a directional vector $\boldsymbol{g} = (g_y, -g_b) = (1, -1)$. This scenario, represented as a movement from point A in direction to point B in figure 2, is expected to occur when undesirable production has costly disposal fees or when restrictions are imposed on production of undesirable output. In the Figure 2, the observation J_1^t jointly produces desirable (*y*) and undesirable (*b*) outputs given an input set (***x***). The directional output distance function seeks to maximize the simultaneous expansion of *y* and contraction of *b*.

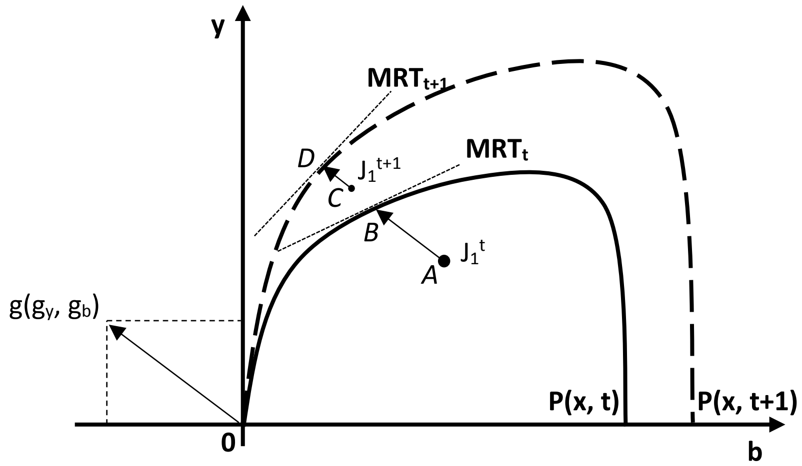

Figure 2: Output Set - *P(x)*, and directional output distance function
Source: Own elaboration.

The distance of the observation with respect to the contemporaneous frontier, also known as inefficiency, is the distance from the frontier (efficient units have a distance equal to zero), given by *B – A* for period *t* and *D – C* for period *t +1.* All efficient units are on the frontier, captured by $\vec{D}_o(x, y, b; 1, -1) = 0$. For all observations in the output set $\vec{D}_o(x, y, b; 1, -1) \geq 0$.

An outwards parallel shift of the frontier suggests neutral progressive technical change while an inwards shift indicates a neutral regressive technical change. A neutral progressive technical change suggests that it is possible to obtain more desirable output and less undesirable output given input levels at the same transformation rate (Marginal Rate of Transformation). A non-neutral technical change is described by a non-parallel shift of the frontier that suggests a change in the transformation between desirable and undesirable outputs. We investigate whether technical change was progressive (regressive) and neutral (non-neutral) or not by focusing on the shift of the output sets described in Figure 2 between *D* and *B*.

3.3.1. Primal output-based directional measure

We evaluate the impact of technical change following the strategy developed by Färe and Karagiannis (2014)[27]. Following Färe and Karagiannis (2014) the total differential of the distance function is

$$-\left(\nabla_b \vec{D}_o\right)' g_b d\alpha + \left(\nabla_y \vec{D}_o\right)' g_y d\alpha + \frac{\partial \vec{D}_o}{\partial t} dt + \frac{\partial \vec{D}_o}{\partial x} dx = 0 \tag{2}$$

Given the definition of technical change we assume $dx = 0$ and a common marginal change in both desirable and undesirable translation factors ($d\alpha$). Solving for technical change and using the translation property[28] it is possible to obtain the rate of technical change as

$$\frac{d\alpha}{dt} = \frac{\partial \vec{D}_o}{\partial t} \tag{3}$$

Färe and Karagiannis (2014) define technical change as the common number of times the desirable output and the undesirable output vectors (g_y and g_b) can be added to the desirable output and subtracted from the undesirable output as a result of technological change. In Figure 2 it is represented by the length of the segment *DB*.

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[27] Badau *et al.* (2016) have applied these concepts.

[28] Translation property implies that the unit will be more efficient by α if an increase on desirable output by α and contraction in undesirable output by α occurs (Färe *et al.*, 2005). Chambers (2002) shows that this can be represented as $-\left(\nabla_b \vec{D}_o\right)' g_b + \left(\nabla_y \vec{D}_o\right)' g_y = -1$.

3.3.2. Bias technical change

There are several ways of investigating technical change biases. Kumar and Managi (2009) uses Antle's (1984) profit-based multifactor measure of biased technical change. We use a primal definition of bias proposed by Fulginiti (2010) and based on *Hicksian* biases of technical change. These are defined as the change in the MRT as a result of technical change along an expansion path

$$B_{mj}(y,b,x,t) \equiv \frac{\partial \ln\left(MRT_{mj}\right)}{\partial t} = \frac{\partial \ln\left(\nabla_{jt}\vec{D}_o / \nabla_{mt}\vec{D}_o\right)}{\partial t}, m,j = 1,\dots,M, m \neq j \tag{4}$$

where m and j represents desirable and undesirable outputs, and $\nabla_{mt}\vec{D}_o$ and $\nabla_{jt}\vec{D}_o$ are derivatives with respect to outputs. Due to monotonicity, this derivative is positive for desirable outputs and negative for undesirable outputs. It measures the biases in technical change as a rotation on the production possibility frontier (*PPF*) in output space (Fulginiti, 2010). $B_{mj} > 0$ suggests that technical change is biased towards the production of the desirable output *m* relative to undesirable output j. $B_{mj} < 0$ indicates that technical change is biased against production of desirable output *m* relative to undesirable output j.

A positive technical change means that more desirable output and less undesirable output is produced after the innovation or technical change. If the bias of this technical change is positive, it is less costly in terms of undesirable goods to increase production of desirable goods.

3.4.THE APPLICATION

Our sample was selected based on the accumulated level of deforestation over the period from 2001 to 2015. We first calculated total deforestation over this period per municipality (more than 700 municipalities in the Legal Amazon region). For example, the municipality of Sao Felix do Xingu, in the state of Para, have deforested one million hectares[29] during the period from 2001 to 2015. On average, each municipality deforested thirty-one thousand hectares over this period. To build our sample we selected municipalities that have total deforestation during this period above this average. Thus, our sample consists of municipalities

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[29] This is equivalent to 5% of Nebraska state area.

that have deforested more than thirty-one thousand hectares during the period 2001-2015. In total, the panel is composed of two hundred municipalities during the period 2003-2015, twenty-six hundred observations.

Desirables outputs and inputs were obtained from the Brazilian Institute of Geography and Statistics (IBGE, 2017). We use data from the Municipal Agricultural Production[30] (PAM) statistics at county level to measure these variables during the period 2003-2015. We have chosen to include in the model grains, livestock and timber production as the main drivers of deforestation. Grains production is the sum of corn and soybean (in tons), timber is measured in cubic meters and livestock in thousand liters of milk. Sold heads would be a better measure of livestock but it is only available for the year of 2006

Deforestation was obtained from the National Institute for Space Research (INPE/PRODES, 2017) and it is measured in hectares. Margulis (2004) suggests that deforestation might occur over three years, and be detected only in the third year of the process, depending on the process of deforestation used. It is possible that agricultural activities would be occurring during this process with revenue from both agriculture and timber sales during this period. This leads us to measure deforestation as the average of the previous three years. Descriptive statistics are in Table 1.

Municipalities in the state of Para and Mato Grosso have shown the largest average deforestation, 8,762 and 6,606 hectares of average deforestation respectively. The average grain production is driven by municipalities in the state of Mato Grosso, they have shown a yearly average production of 248,806 tons of grains. On milk production, municipalities in the state of Rondônia have shown the largest average, of 10,078 thousand liters of milk. Municipalities in the state of Para have shown the largest average production of timber, 67,951 m^3. In Appendix 3.A we display the geographical distribution of these activities.

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[30] The Agricultural Census is only available for the year of 2006.

Table 1. Descriptive Statistics for Agricultural Outputs, Inputs and Deforestation in 200 Municipalities during the period from 2003-2015 in the "arc of deforestation", Brazil.

	Variable	Mean	Standard Deviation	Min.	Max.
Outputs					
Average Deforestation (ha)	b_1	6144.9	9996.3	0.0	142463.3
Grains (tons)	y_1	79099.8	299309.6	0.0	4584870.0
Milk (1000 litters)	y_2	5539.1	8313.4	0.0	91953.0
Timber (m^3)	y_3	50946.9	129764.5	0.0	1521233.0
Inputs					
Labor (units)	x_1	40237.5	83333.6	1225.0	1064197.0
Capital (units)	x_2	199093.3	223584.2	0.0	2282445.0
Public Expenditure (R$ 1000)	x_3	531346.8	1130518.0	0.0	13700000.0
Agricultural area (ha)	x_4	419933.8	411943.0	420.0	4269020.0

Source: Desirable outputs and inputs were obtained at SIDRA/IBGE and deforestation at INPE/PRODES.

We were able to obtain information on four inputs. Labor is represented by municipality population. On average, fifty-one percent of the municipality's population are in rural areas; less than twenty-five percent of these municipalities have rural population less than thirty percent. Agricultural area is measured in hectares and it was built subtracting total municipality area by forest area each year. Capital has been represented in the literature by stock of livestock. We use this approach to measure capital in cattle numbers. We also included public expenditure on agriculture (in R$ 1000) seeking to capture extension[31] as an input. The first three inputs were obtained at IBGE while the last input at the Brazilian National Treasure Secretary

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[31] This expenditure accounts to extension support, livestock and crop promotion and irrigation support. This series was deflated using the IGP-DI index available at the Institute for Applied Economic Research (IPEA, 2017).

(2017)[32]. Following Färe *et al.* (2005), all these variables are normalized by their means[33], aiming to achieve convergence in the stochastic estimation. In addition to these inputs we add a time trend to capture exogenous technical change.

3.4.1. Empirical strategy

We approximate the distance function (Eq. 1) using a quadratic functional form, with the subscript $i = (1,2,\dots,N)$ representing municipalities, subscript *t* dropped for simplicity

$$\begin{aligned}\vec{D}_{o,i}(x,y,b,;t,1,-1) = {} & \gamma_0 + \sum_{k=1}^{4}\gamma_k x_{ki} + \theta_1 b_i + \sum_{m=1}^{3}\beta_m y_m + \frac{1}{2}\sum_{k=1}^{4}\sum_{l=1}^{4}\gamma_{kl}x_{ki}x_{li} \\ & + \frac{1}{2}\sum_{m=1}^{3}\sum_{m'=1}^{3}\beta_{mm'}y_{mi}y_{m'i} + \frac{1}{2}\theta_{11}{b_i}^2 + \sum_{m}^{3}\sum_{k=1}^{4}\delta_{mk}x_{ki}y_{mi} + \sum_{k=1}^{4}\varphi_k x_{ki}b_i \qquad (5) \\ & + \sum_{m=1}^{3}\mu_m y_{mi}b_i + \upsilon_1 t + \frac{1}{2}\upsilon_{11}t^2 + \sum_{k=1}^{4}\vartheta_{k1}x_{k,i}t + \sum_{m}^{3}\eta_m y_{mi}t + \lambda_1 t b_i\end{aligned}$$

where x_{ki} are labor, capital, public expenditures in agriculture and area, y_{mi} are timber, milk and grains, b_i is deforestation, *t* is technical change and γ_0, β's, θ's, δ's, φ's, υ's, ϑ's and μ's are parameters to be estimated. The constant is composed by a constant term and municipality fixed effects (dummies). As Färe *et al.* (2005) we use the directional vector $\boldsymbol{g} = (g_y, -g_b) = (1,-1)$ representing a simultaneous expansion in desirable outputs and a contraction of undesirable outputs. The symmetry and translation property in outputs are imposed before estimation, requiring the following restrictions

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[32] Specifically, we use the dataset available online named Financas do Brasil (FINBRA, 2017).

[33] For a hypothetical municipality that uses mean inputs and produces mean outputs, the input and output variables would be $(x,y,b) = (1,1,-1)$. It implies that Figure 1 is in normalized values. Thus observation J1 (illustrated in Figure 2) can be expanded by $\alpha^*\bar{y}$, and contracted by $\alpha^*\bar{b}$ simultaneously.

$$\sum_{m}^{3}\beta_{m}-\theta_{1}=-1;\quad \sum_{m'=1}^{3}\beta_{mm'}-\mu_{m}=0;\quad \theta_{11}-\sum_{m=1}^{3}\mu_{m}=0;\quad \sum_{m}^{3}\delta_{mk}-\varphi_{k}=0$$

$$\sum_{m}^{3}\eta_{m}-\lambda_{1}=0,\ \text{m =1, 2 and 3; k =1, 2, 3 and 4};\quad \beta_{gf}=\beta_{fg}$$

We estimated equation (5) after imposing these properties as

$$-\alpha_i=\vec{D}_{o_i}\left(x_i, y_i+\alpha_i, b_i-\alpha_i; 1,-1\right)+\ \epsilon_i, \tag{6}$$

where α_i is the translation factor. The following quadratic functional form with symmetry and translation properties imposed is estimated

$$\begin{aligned}-b_i=\ &\gamma_0+\sum_{k=1}^{4}\gamma_k x_{ki}+\theta_1 b'_i+\sum_{m=1}^{3}\beta_m y'_m+\frac{1}{2}\sum_{k=1}^{4}\sum_{l=1}^{4}\gamma_{kl}x_{ki}x_{li}\\ &+\frac{1}{2}\sum_{m=1}^{3}\sum_{m'=1}^{3}\beta_{mm'}y'_{mi}y'_{m'i}+\frac{1}{2}\theta_{11}{b'_j}^2+\sum_{m}^{3}\sum_{k=1}^{4}\delta_{mk}x_{ki}y'_{mi}+\sum_{k=1}^{4}\varphi_k x_{ki}b'_i\\ &\quad+\sum_{m=1}^{3}\mu_m y'_{mi}b'_i+\upsilon_1 t+\frac{1}{2}\upsilon_{11}t^2+\sum_{k=1}^{4}\vartheta_{k1}x_{k,i}t+\sum_{m}^{3}\eta_m y'_{mi}t+\lambda_1 t b'_i\end{aligned} \tag{7}$$

where $y'_{1i}=y_{1i}+\alpha_i$, $b'_i=b_i-\alpha_i$. In our case, $\alpha_i = b_i$[34], then the parameters associated to b_i are obtained after estimation using the translation property restrictions. Technical change is estimated following equation (3) as

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[34] Summary and Weber (2012) and Weber and Xia (2011) have considered as the translation factor the negative of a desirable output (i.e. $\alpha_i = -y_{1i}$), since they only have desirable outputs. Kumar and Managi (2010) include undesirable outputs in their analysis and have used as the translation factor a desirable output (i.e. $\alpha_i = y_{1i}$). These specifications would change equation (6).

$$\frac{\partial \vec{D}_o}{\partial t} = \upsilon_1 + \upsilon_{11} t + \sum_{k=1}^{4} \vartheta_{k1} \mathrm{x}_{\mathrm{k,i}} + \sum_{m}^{3} \eta_m y_{mi} + \lambda_1 b_i \tag{8}$$

Technical change biases are calculated using Equation (4) as

$$B_{y_{mi}, b_i}(y_{mi}, b_i, x, t) \equiv \left[\frac{\lambda_1}{\nabla \vec{D}_{b_i}} - \frac{\eta_m}{\nabla \vec{D}_{y_{mi}}} \right] \tag{9}$$

where $\nabla \vec{D}_{b_i}$ and $\nabla \vec{D}_{y_{mi}}$ represent the first derivative of the directional distance function with respect to the undesirable and desirable outputs, respectively, or

$$\begin{aligned} \nabla \vec{D}_{b_i} &= \theta_1 + \theta_{11} b + \sum_{\mathrm{r}=1}^{4} \varphi_{\mathrm{k1}} \mathrm{x}_{\mathrm{ki}} + \sum_{m=1}^{3} \mu_m y_{mi} + \lambda_1 t \geq 0 \\ \nabla \vec{D}_{y_{mi}} &= \beta_m + \sum_{m=1}^{3} \beta_{mm'} y_m + \sum_{\mathrm{k}=1}^{4} \delta_{\mathrm{k1}} \mathrm{x}_{\mathrm{ki}} + \mu_{\mathrm{m1}} \mathrm{b} + \eta_m t \leq 0 \end{aligned} \tag{10}$$

by the monotonicity property, i.e. the directional distance should not decrease with undesirable outputs and not increase with desirable outputs. These properties will be cheeked after estimation.

Equation (7) was estimated using Maximum Likelihood Estimation (MLE)[35]. We first estimate this equation using Corrected Ordinary Least Squares (COLS) to use its estimated parameters as starting values for the MLE procedure. The inefficiency is estimated as $E(u_i|(\epsilon_i = u_i - z_i))$. The MLE does not incur on parameter biases as the COLS given that the random error $z_i \sim i.i.d.N(0, \sigma_z^2)$. The estimation was done using Stata 14 following the command *sfmodel* suggested by Kumbhakar *et al.* (2015) and *sfcross* suggested by Belotti *et al.* (2012). Our empirical strategy does not impose the properties of monotonicity and concavity.

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[35] Appendix II, located at the end of the thesis, describes these methods more extensively in addition to guide you to the references on these methodologies.

3.5.RESULTS AND DISCUSSION

We estimated the quadratic specification for the directional distance function in equation (7) using MLE. Parameters estimated are displayed in Table2. The MLE estimation has thirty-seven statistically significant parameters out of 45 (excluding municipality dummies). On the theoretical properties[36,37], monotonicity property was checked at each data point after estimation; less than 2% of the observations violate monotonicity in grains, around 9% violate it with respect to timber, less than 2% with respect to livestock and less than 4% with respect to deforestation. A Likelihood Ratio test of 19.85 indicates that MLE estimates with a half-normal distribution for the one-sided error term are superior to the COLS estimates at one percent (critical value is 5.4).

Table 2 – MLE Parameter Estimates for Directional Distance Function in Municipalities of the "arc of deforestation", Brazil in from 2003-2015

Coefficient	*Variable*	*Parameter*	*Standard Error*
$\boldsymbol{\beta_1}$	y_1	-0.2071***	(0.0131)
$\boldsymbol{\beta_2}$	y_2	-0.4904***	(0.0130)
$\boldsymbol{\beta_3}$	y_3	-0.1104***	(0.0086)
$\boldsymbol{\beta_{11}}$	y_1^2	0.0016***	(0.0004)
$\boldsymbol{\beta_{22}}$	y_2^2	0.0614***	(0.0029)
$\boldsymbol{\beta_{33}}$	y_3^2	0.0072***	(0.0006)
$\boldsymbol{\beta_{12}}$	y_1y_2	-0.0485***	(0.0024)
$\boldsymbol{\beta_{13}}$	y_1y_3	0.0165***	(0.0019)
$\boldsymbol{\beta_{23}}$	y_2y_3	-0.0124***	(0.0019)
$\boldsymbol{\gamma_1}$	x_1	-0.0368	(0.0614)
$\boldsymbol{\gamma_2}$	x_2	0.4629***	(0.0453)
$\boldsymbol{\gamma_3}$	x_3	-0.0312***	(0.0102)

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[36] Our estimate of the directional distance function is not concave on *y* and *b*. Although non-concavity is not desirable the violation of this property influences the analysis of the change in the slope within outputs second derivatives. Specifically, it affects analysis that depend directly of the second order coefficients associated with the outputs, $\beta_{mm'}$. For example, Färe *et al.* (2012) has estimated the output elasticity which depends directly of these coefficients. Our analysis does not depend of those directly.

[37] Less than 25% of the observations violate null-jointness hypothesis.

$\boldsymbol{\gamma_{4}}$	x_4	-0.0927**	(0.0421)
$\boldsymbol{\gamma_{11}}$	x_1x_1	0.0222***	(0.0057)
$\boldsymbol{\gamma_{22}}$	x_2x_2	-0.0936***	(0.0206)
$\boldsymbol{\gamma_{33}}$	x_3x_3	-0.0005	(0.0010)
$\boldsymbol{\gamma_{44}}$	x_4x_4	0.0228***	(0.0075)
$\boldsymbol{\gamma_{12}}$	x_1x_2	-0.0115	(0.0114)
$\boldsymbol{\gamma_{13}}$	x_1x_3	-0.0005	(0.0006)
$\boldsymbol{\gamma_{14}}$	x_1x_4	0.0477***	(0.0183)
$\boldsymbol{\gamma_{23}}$	x_2x_3	-0.0194***	(0.0047)
$\boldsymbol{\gamma_{24}}$	x_2x_4	-0.0795***	(0.0186)
$\boldsymbol{\gamma_{34}}$	x_3x_4	0.0064**	(0.0031)
$\boldsymbol{\delta_{11}}$	y_1x_1	-0.0426***	(0.0044)
$\boldsymbol{\delta_{12}}$	y_1x_2	-0.0325***	(0.0069)
$\boldsymbol{\delta_{13}}$	y_1x_3	-0.0018***	(0.0006)
$\boldsymbol{\delta_{14}}$	y_1x_4	0.0414***	(0.0054)
$\boldsymbol{\delta_{21}}$	y_2x_1	0.0316***	(0.0057)
$\boldsymbol{\delta_{22}}$	y_2x_2	-0.0031	(0.0070)
$\boldsymbol{\delta_{23}}$	y_2x_3	0.0106***	(0.0030)
$\boldsymbol{\delta_{24}}$	y_2x_4	-0.0298***	(0.0078)
$\boldsymbol{\delta_{31}}$	y_3x_1	-0.0062***	(0.0021)
$\boldsymbol{\delta_{32}}$	y_3x_2	0.0100***	(0.0033)
$\boldsymbol{\delta_{33}}$	y_3x_3	-0.0023**	(0.0011)
$\boldsymbol{\delta_{34}}$	y_3x_4	-0.0007	(0.0027)
$\boldsymbol{\upsilon_{1}}$	t	0.0158***	(0.0060)
$\boldsymbol{\upsilon_{11}}$	t^2	0.0001	(0.0009)
$\boldsymbol{\vartheta_{11}}$	x_1t	-0.0083***	(0.0015)
$\boldsymbol{\vartheta_{21}}$	x_2t	0.0106***	(0.0022)
$\boldsymbol{\vartheta_{31}}$	x_3t	0.0049***	(0.0009)
$\boldsymbol{\vartheta_{41}}$	x_4t	-0.0048***	(0.0019)
$\boldsymbol{\eta_{11}}$	y_1t	0.0109***	(0.0008)
$\boldsymbol{\eta_{21}}$	y_2t	0.0045***	(0.0012)
$\boldsymbol{\eta_{31}}$	y_3t	0.0030***	(0.0007)

γ_0	*Constant*	-0.0576	(0.0918)
σ_u		-2.7948***	(0.2075)
σ_v		-3.7911***	(0.1767)
λ_{MLE}		0.2472***	(0.0256)

Note: COLS parameters used as starting values for MLE. Standard error in parenthesis; *** for p-value smaller than 0.01, ** smaller than 0.05, and * smaller than 0.1. The dependent variable is negative of average deforestation.

λ_{MLE} refers to the estimated σ_u/σ_v (see Appendix II) instead of the parameter associated to the interaction between undesirable output and time trend (Eq. 7). It includes municipality dummies. Parameters for the translation factor, deforestation, are recover using these parameters and the translation property. In total, 2600 observations were used to estimate this regression.

An estimate of the distance of each municipality from the frontier is obtained from equation (7), and is interpreted as a measure of inefficiency. The average distance estimated for the region was 0.19. This means that, on average, agricultural outputs (grains, timber and livestock) could be expanded by 19% each while simultaneously decreasing deforestation by 19%.

Our first objective was to identify whether technical change was progressive or regressive, that is if the PPF has shifted outwards or inwards. The rate and biases vary over the production space, depending on the level of inputs and outputs for individual counties at each point in time. We estimated average technical change in three ways. First taking the simple average of the technical change rates, which will be present in the body of the text (Table 3). Second, we used a weighted average approach where the weights are the share of the value of production per state (and per region). These results are presented in the Appendix 3.B. This weighting strategy considered only agricultural production as a weight factor and did not include any measure of deforestation. Therefore, we only present them on the Appendix. Third, we have evaluated Equation (8) on the average of all outputs (desirable and undesirable) and inputs. Technical change rate using this approach is 5.4%.

The average rate of technical change estimated for this region during the period 2003-2015 is 4.58%,[38,39]. It means that, on average, technical change has allowed municipalities to expand

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[38] Standard error and *p-value* were estimated for these estimates. State averages were statistically significant at least at 10%.

[39] Technical change rates presented in this section were calculates taking the simple average of the evaluation of Equation (8) for every observation in the sample.

agricultural outputs (grains, timber and livestock) by around 4.6% while simultaneously contracting deforestation by 4.6%. Table3 displays theses estimates per state over the period from 2003 to 2015.

Table 3 – Average rates of technical change (in percentage) for municipalities in the "arc of deforestation" in states in the Legal Amazon, Brazil, 2003-2015 [evaluation of Eq. (8)].

Year	**RO**	**AC**	**AM**	**RR**	**PA**	**MA**	**MT**	**Yearly Average**
2003	5.65	4.34	4.02	2.95	6.85	6.77	7.24	**6.40**
2004	6.06	4.59	4.45	2.38	5.78	1.94	8.13	**6.40**
2005	6.33	4.23	4.86	2.48	5.94	1.83	7.87	**5.49**
2006	5.98	4.30	4.37	1.88	5.76	1.82	7.02	**5.42**
2007	5.79	3.91	3.66	2.82	4.94	1.73	6.15	**5.11**
2008	5.24	3.56	3.56	3.18	5.14	1.78	5.63	**4.61**
2009	4.85	3.78	3.12	2.97	4.89	1.82	5.51	**4.43**
2010	4.70	3.24	2.79	2.67	4.72	1.87	5.39	**4.26**
2011	4.84	4.02	2.73	2.06	4.17	1.73	5.23	**4.12**
2012	5.61	4.16	2.96	2.57	4.17	1.59	5.34	**3.93**
2013	5.32	3.87	3.53	2.03	3.82	1.55	5.36	**3.99**
2014	5.44	3.58	3.59	2.18	3.96	1.59	5.58	**3.84**
2015	5.37	3.27	3.95	2.19	3.90	1.56	5.60	**3.91**
State Average	**5.47**	**3.92**	**3.66**	**2.51**	**4.91**	**2.12**	**6.23**	**4.58**

The regional average is driven by municipalities in three states on the outer boundary of the Amazon Forest: Mato Grosso, Rondônia and Para. They have shown a state average rate of technical change for the entire period of 6.23%, 5.5% and 4.9%, respectively. Figure 3 displays these results graphically and Figure 4 presents their geographical distribution for 2012.

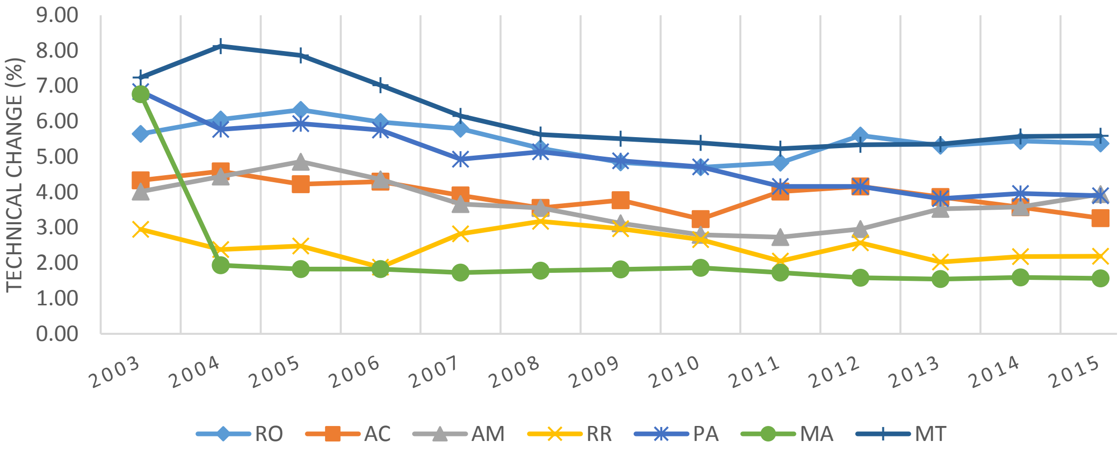

Figure 3 –Average technical change rates by state in the “arc of deforestation”, northern region of Brazil, from 2003 to 2015.

The lower boundary of the “arc of deforestation” (or the outer boundary of the Amazon Forest), where agricultural production has been established for several years, has municipalities with high rates of technical change. Analysis of Figure 4 indicates that municipalities with the highest rates of technical change are clustered in the south and southeast part of the “arc of deforestation”. They are located in the state of Mato Grosso and in the southeast of the state of Para. Some of these municipalities have had high rates of deforestation.

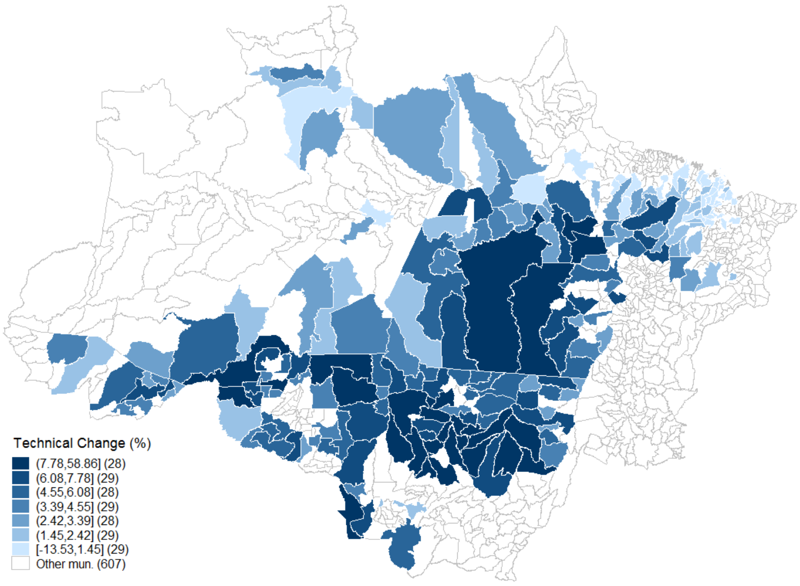

Figure 4 – Stochastic municipal technical change rate in the “arc of deforestation” in the northern region of Brazil, in 2012.

In 2007, the Brazilian government created a list of priority municipalities, which have a high level of deforestation. They are spread in the "arc of deforestation" region[40] but clustered on Para and Mato Grosso, as seen in Figure C31 in Appendix 3.C. Given the government efforts in controlling deforestation in these municipalities, we perform a test of means of technical change rates comparing this set of municipalities with the other municipalities in the "arc of deforestation". The test[41] suggests that priority municipalities have shown a higher technical change rate than the other municipalities, probably due to a sharper decrease on deforestation.

Figure 5 displays the evolution of agricultural output, deforestation as well as the estimated average rates of technical change. It suggests that the technical change has led to increase in agricultural outputs (mainly grains) and decrease in deforestation for the entire region.

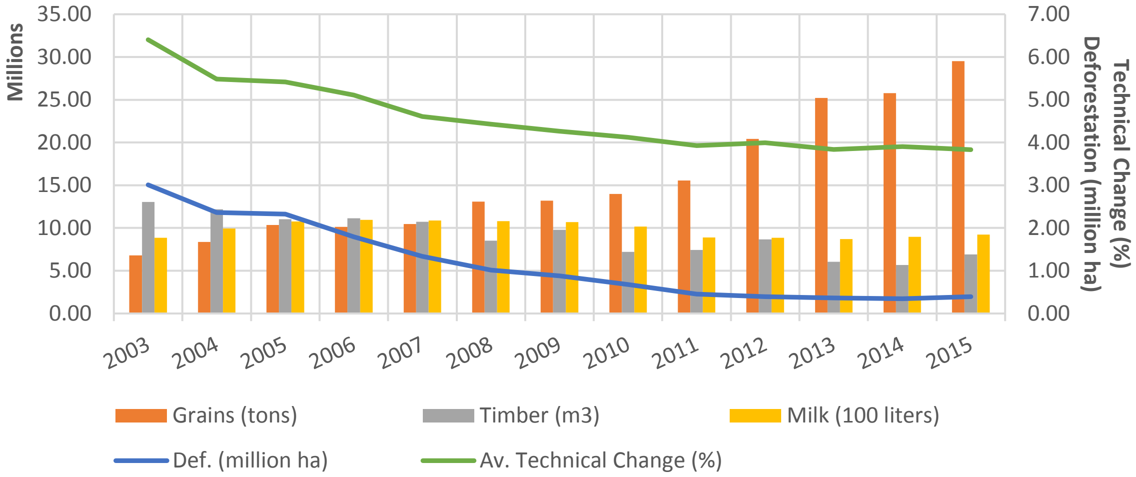

Figure 5 – Evolution of agricultural output, deforestation and the average rate of technical change for the "arc of deforestation" in the northern region of Brazil.

Note: Grains (in tons), timber (in m^3) and milk (in 100 liters) vertical axis is displayed at left axis while deforestation (in million hectares) and technical change (in percentage) on the axis at the right.

The exogenous technical change rate may have been affected by policies during this period. Brazilian government has worked in the two fronts in this region: controlling deforestation and promoting agricultural production. Deforestation control in these municipalities has been

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[40] The 49 priority municipalities appointed by INPE/PRODES are within the sample we selected in this paper.

[41] The null hypothesis of no difference in their mean was rejected at 1%. The priorities municipalities have an average technical change of 0.078 while the other have an average of 0.048. The calculated value for the test was -5.44. this test was performed using Stata 14.

enforced and voluntary applied by different institutions. Brazilian government launched in 2004 the Action Plan for Deforestation Prevention and Control in the Legal Amazon[42] seeking to control the deforestation in the Legal Amazon region. Nepstad *et al.* (2014) suggest that this program increased governance performance and allowed a wider forestry monitoring and enforcement of the policy, which led to a decrease in deforestation on the following years.

Interventions such as the Soy Moratorium (SoyM) in 2006, and the Cattle Agreement in 2010, constituted obstacles to deforestation despite the fact that neither are enforced, but instead are voluntary policies (Nesptad *et al.*, 2014; Gibbs *et al.*, 2014). The enforcement of new regulations such as the Brazilian Forest Code (FC), the Rural Environmental Registry of private properties (CAR), and surveillance by Brazilian Institute of the Environment and Renewable Natural Resources (IBAMA), have had positive impacts as deforestation control mechanisms (Gibbs *et al.*, 2014; Soares-Filho *et al.*, 2014; Hargrave and Kis-Katos, 2013).

On the other hand, the Brazilian government has invested on infrastructure, public research and promoted agricultural production via increasing credit availability and extension support (Gomes and Braga (2008); Gasques *et al.* (2014)). From 1999 to 2009, credit availability through the Program to Support the Family Farming (PRONAF)[43] has increased yearly at a rate of 23% for the seven states considered in this paper. The total credit made available by the government to this region was six-fold in 2009 compared to 2001. Additionally, total municipal public expenditure on agriculture in this region, x_3, in 2015 was almost twice 2003 expenditure level. These policies certainly affected agricultural production and forest preservation and may have influenced how innovations take place on agriculture in this region.

3.5.1. Technical change biases

Our second objective was to verify whether technical change was biased for agricultural production and against deforestation or vice-versa. We evaluate this assertion by estimating how the Marginal Rate of Transformation between agricultural outputs and deforestation has changed after technical change took place. Our results as derived below indicate that municipalities in the "arc of deforestation" have been experiencing biased technical change in

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[42] Plano de Ação para a Prevenção e Controle do Desmatamento na Amazônia Legal – PPCDAm found at http://www.mma.gov.br/florestas/controle-e-preven%C3%A7%C3%A3o-do-desmatamento/plano-de-a%C3%A7%C3%A3o-para-amaz%C3%B4nia-ppcdam

[43] Programa Nacional de Fortalecimento da Agricultura Familiar

favor of agricultural output and against deforestation. This means that, after the technical change took place, the cost of increasing agricultural output in terms of deforestation has decreased. That to increase agricultural output, less deforestation is necessary.

We evaluate technical change bias relative to deforestation using Equation (9), for each subset of desirable outputs. For example, to evaluate whether technical change has been bias toward grains production and against deforestation we evaluate Equation (9), B_{y_1,b_1}, for all observations. We proceed in the same manner when evaluating the technical biases with respect to timber and livestock. Then, we take a simple average of these pairwise biases to represent the bias. We have found that the simple averages of the regional biases for grains-deforestation was $B_{y_1,b_1} = 0.19$, of timber-deforestation was $B_{y_3,b_1} = 0.21$ and of milk-deforestation was $B_{y_2,b_1} = 0.15$. These estimates indicate that more of these agricultural outputs have to be foregone to decrease one hectare of deforestation. It suggests that due to bias technical change, abatement costs are more costly and increases in grain output now require less deforestation.

3.6.CONCLUSIONS

This paper evaluates whether the high rates of technical change reported in the literature for Amazon agriculture stand when deforestation, an undesirable output, is consider in the calculation. It also evaluates the nature of the biases in technical change under these circumstances to understand if innovations have made it less or more costly in terms of deforestation to increase agricultural production. Our sample is composed by two hundred municipalities in the "arc of deforestation" in Brazil over the period 2003-2015. An aggregate municipality-level technology was estimated using a directional distance function with data from the IBGE and the INPE. The directional distance function was approximated by a flexible quadratic form and estimated using a stochastic approach.

Our results reveal that technical change have led to an increase agricultural production and a decrease in deforestation during the period from 2003 to 2015. Municipalities in Mato Grosso state has shown the highest average technical change rate, 6.2%, while the overall average was 4.6%. On average, these estimates are lower than the 8.5% for this state reported by Bragnanolo *et al.* (2010) for the period 1975-2006. Villoria *et al.* (2014) highlights the lack of empirical studies investigating the effects of technical change in agriculture on forest preservation. Our results suggest that innovation in the sector have allowed increased agricultural production with less deforestation. The estimated biases indicate that an increase in agricultural production is now possible with a smaller increase in deforestation.

Although deforestation has been decreasing and it has achieved the lowest level in 2015 compared to entire period, it is still a prominent activity in these municipalities. New numbers have shown an increase in deforestation in 2016 compared to 2015; the state of Mato Grosso has shown an increase in deforestation of 190% during the first months of 2016 compared to 2015[44]. Future research will focus on imposition of all the regularity properties of the technology as they will impact our estimates. We would also benefit from a dataset such as the Ag. Census for the years 1975, 1985, 1995 and 2006 made available by IBGE on-site.

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[44] http://g1.globo.com/mato-grosso/noticia/2016/05/desmatamento-da-amazonia-legal-aumenta-190-em-mt-diz-imazon.html

FINAL REMARKS

Preserving a hectare of Amazon forest brings many benefits – it sequesters carbon and thus reduces potential climate change, it contributes to preservation of many plant and animal species with unknown value, and people value its existence, per se. However, these are benefits to mankind as a whole, while the opportunity costs of preservation fall upon the Brazilians themselves, who must trade off potential agricultural income forgone in exchange for a hectare of forest preserved. This work uses a new approach and new data to provide new estimates of this opportunity cost in the Brazilian Amazon. It also evaluates the effects of technical change on this tradeoff. It does so by estimating the aggregate municipality-level technology using directional distance functions. Three different datasets and two estimation approaches were used to accomplished these goals.

We have found that, on average, US\$796.81 in agricultural Gross Domestic Product has to be foregone annually to preserve one additional hectare of forest. This translates to an average shadow price of US\$21.7 per ton of CO_2 sequestered in perpetuity (at a social discount rate of 0.10 and 100 tons per hectare of carbon sequestered). Our estimates of the opportunity cost of a ton of CO_2 are higher than what has been used by the Amazon Fund to raise funds to preserve the Amazon Forest. These estimates do depend on the assumption on carbon content, discount rate and length of the period considered.

Technical change has been progressive in this region over the period from 2003 to 2015. It means that innovations have led to agricultural expansion with less deforestation. Technical change has been biased toward grains, timber and livestock outputs and against deforestation. Strengthening agricultural research and extension efforts in a policy environment that leads to forest preservation could result in increases in agricultural production with lower increases in deforestation. It is also the case that if regulation were put in place to decrease deforestation abatement costs will be higher.

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Appendix 1.A – Data description

Table A31: Desirable and undesirable outputs – livestock, grains and timber production, and deforestation per state for the Arc of deforestation in the Brazilian Amazon region in 2006, average and sum across municipalities.

State	# of Munic.		Livestock (units)	Grains (tons)	Timber (m^3)	Deforestation (*ha*)
RO	24	*Mean*	29402	3645	61370	31307
		Sum	705655	87490	1472880	751370
AC	6	*Mean*	27236	7526	53382	22120
		Sum	163415	45157	320292	132720
AM	6	*Mean*	11720	353	50203	23465
		Sum	70321	2115	301217	140790
PA	50	*Mean*	37895	5056	196487	40201
		Sum	1894766	252785	9824337	2010050
TO	1	*Mean*	16305	1298	38	10640
		Sum	16305	1298	38	10640
MA	11	*Mean*	17858	7616	7272	19510
		Sum	196433	83771	79991	214610
MT	58	*Mean*	41171	108156	49565	32498
		Sum	2387932	6273039	2874750	1884880
ARC	156	***Mean***	**34839**	**43241**	**95343**	**32981**
		Sum	**5434827**	**6745655**	**14873504**	**5145060**

Source: Data obtained at IBGE (2014).

Note: Deforestation (ha) is the average deforested area occurred during the period 2004-2006.

Table A32: Sum of inputs (labor, capital, area, fuel, agricultural inputs, and livestock inputs) per state for the Arc of deforestation in the Brazilian Amazon region in 2006

State		Labor (unit)	Capital (unit)	Area (ha)	Fuel (US$)	Ag. Inputs (US$)	Livest. Inputs (US$)
RO	*Mean*	6227	2976	211823	1254	692	3504
	Sum	149437	71424	5083745	30102	16616	84090
AC	*Mean*	5433	2904	272016	981	240	2331
	Sum	32595	17426	1632095	5886	1442	13986
AM	*Mean*	5541	1673	210053	414	91	1036
	Sum	33245	10038	1260316	2483	545	6218
PA	*Mean*	7583	2666	327891	1372	792	3549
	Sum	379172	133320	16394552	68619	39594	177452
TO	*Mean*	2946	1408	119137	589	2666	2972
	Sum	2946	1408	119137	589	2666	2972
MA	*Mean*	5810	2264	181106	735	502	3060
	Sum	63908	24907	1992164	8082	5522	33659
MT	*Mean*	3298	2350	495469	3649	24729	5464
	Sum	191261	136292	28737186	211653	1434271	316930
ARC	***Mean***	**5465**	**2531**	**353969**	**4554**	**20875**	**8837**
	Sum	**852564**	**394815**	**55219195**	**327415**	**1500657**	**635306**

Source: Data obtained at IBGE (2014)

Note: Capital is the sum of machinery; Fuel is the sum of expenses with electricity and fuel; Agricultural inputs is the sum of expenses with seed, pesticides and fertilizer; and Livestock inputs is the sum of expenses with feed, medication and animals. All variables signed with a US$ are in thousand dollars.

Appendix 1.B – Different models

These tradeoff measures are described in the set of equations (4). First, we compare the quantities (each separately) obtained by projecting to the frontier with different directional vectors $(g_1, g_2, g_3, g_b) = (0,1,0,0)$ for livestock for example, represented by the model C in Table 1, with the quantities obtained by projecting along the vector $(g_1, g_2, g_3, g_b) = (0,0,0,-1)$, represented by the model E in Table 1. In figure 2, this comparison is represented by the vertical distance from point g to point *f*. Table B1 also displays the comparison between quantities in the frontier from model C and D. The difference between these two quantities gives an idea of how much desirable output the municipality has to forego to achieve the largest reduction in undesirable output possible without decreasing the other two desirable outputs from their 2006 level. These measures represent a hypothetically case where we could decrease deforestation by only affecting one agricultural activity.

Table B1 reports the average of these reductions in output and due to decrease on deforestation for municipals within each state. It suggests that more than eight sold heads have to be foregone to reduce one hectare of deforestation, while a smaller ratio is observed for tons of grains (7.38) and a higher ratio for m^3 of timber (597.60). As expected, this ratio is smaller when comparing models C and D (or the vertical distance from g and h) since in this case an expansion on desirable outputs is also allowed for model D.

Table B31 – Tradeoff for several models, obtained using equation (4), in quantity (heads, tons and cubic meters) per hectare of forest (deforestation)

State	Model C – Model E			Model C – Model D		
Region	**Liv/Def**	**Gr/Def**	**Tim/Def**	**Liv/Def**	**Gr/Def**	**Tim/Def**
RO	6.97	6.76	601.43	2.12	5.46	596.58
AC	13.52	11.09	976.76	6.55	7.25	972.64
AM	5.98	4.24	381.00	1.78	2.48	367.90
PA	6.21	4.94	471.22	1.78	3.06	465.80
TO	17.63	39.05	1240.41	11.17	27.67	1238.09
MA	10.20	7.44	815.90	6.67	4.42	880.21
MT	10.57	10.17	697.55	8.10	6.83	686.76
Region	**8.21**	**7.38**	**597.60**	**4.50**	**5.42**	**593.25**

Note: “Liv/Def” represents Sold Heads per Deforestation (ha), “Gr/Def” represents Grains (tons) per Def. (ha), and “Tim/Def” represents Timber (m3) per Def. (ha).

Source: Own elaboration.

In the Brazilian Amazon region one sold head, on average, is produced within half hectare, a ton of grain needs half of hectare, and one hectare of forest produces a range from 25m^3 to 70m^3 of timber (Vera-Diaz and Schwartzman, 2005). Thus, in terms of agricultural activity area, using the analysis in Table 4, a reduction of one hectare of deforestation would have a more severe effect on cattle activity when only one activity is affected, given that would be necessary to decrease around 12 hectares of pasture.

Appendix 2.A – Data description

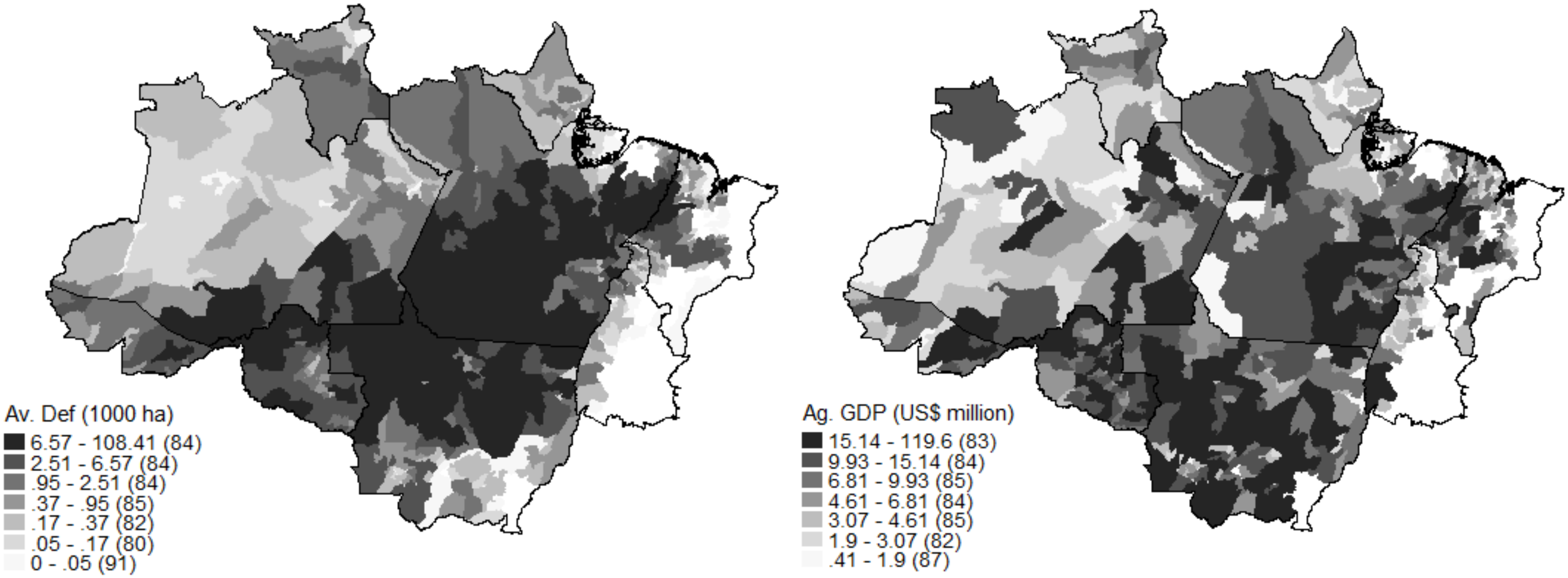

Figure A1. Average deforestation (during 2004-2006 period) and agricultural GDP in 590 municipalities in the Legal Amazon, Brazil in 2006

Source: Own elaboration with data from Prodes/INPE and agricultural Census 2006/IBGE.

Table A21. State Level Descriptive Statistics: Agricultural GDP, Inputs and Deforestation in 590 Municipalities in the Legal Amazon, Brazil in 2006

		RO	**AC**	**AM**	**RR**	**PA**	**AP**	**TO**	**MA**	**MT**	**LA**
Municipals	*Sum*	52	22	62	15	104	16	69	124	126	**590**
Ag. GDP	*Mean*	11818	9189	7266	4727	8180	3157	3420	6025	15922	**8862**
(US$1000)	*Sum*	614553	202161	450519	70907	850767	50508	236010	747091	2006185	**5228703**
Deforest.	*Mean*	5663	2709	1470	1454	6895	438	280	832	5677	**3438**
(hectares)	*Sum*	294493	59590	91147	21813	717100	7000	19287	103193	715330	**2028953**
Capital	*Mean*	2646	1960	1142	773	2088	252	660	1406	2080	**1638**
(Sum of equip.)	*Sum*	137582	43113	70804	11595	217136	4025	45532	174358	262062	**966207**
Labor	*Mean*	5341	4526	4301	1967	6340	818	1322	4084	2700	**7544**
(Sum of employees)	*Sum*	277757	99579	266667	29509	659336	13095	91192	506440	340213	**2283788**
Irrigation	*Mean*	272	66	99	848	197	142	310	321	1005	**415**
(hectares)	*Sum*	14119	1446	6119	12721	20467	2279	21368	39765	126615	**244899**
Credit	*Mean*	1252	457	237	666	1181	142	752	498	8782	**2449**
(US$ 1000)	*Sum*	65088	10044	14709	9997	122853	2272	51898	61744	1106485	**1445090**

Appendix 3.A – Data display

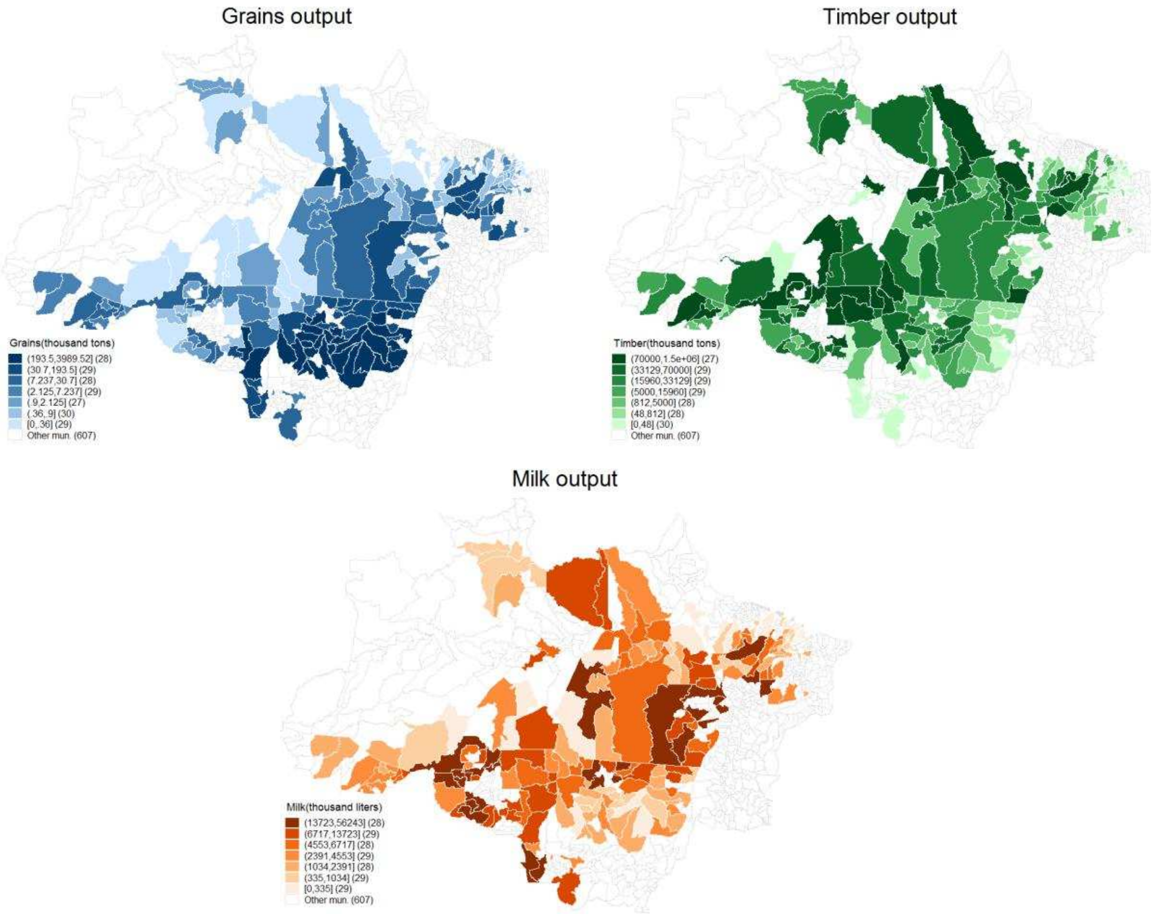

Figure A31 – Grains, timber and livestock (milk) output in the "arc of deforestation" in the northern region of Brazil.

Source: Own elaboration using Stata 14.

Appendix 3.B – Weighted technical change rates

Table B31. Stochastic estimates of the weighted state average of technical change rate for the municipalities in the "arc of deforestation" in Brazil from 2003-2015 [evaluation of Eq. (8)].

Year	RO	AC	AM	RR	PA	MA	MT	Yearly Average
2003	0.070	0.045	0.038	0.033	0.120	0.051	0.180	0.154
2004	0.081	0.047	0.039	0.028	0.116	0.030	0.194	0.169
2005	0.094	0.042	0.044	0.027	0.112	0.032	0.174	0.150
2006	0.092	0.045	0.040	0.021	0.105	0.033	0.164	0.136
2007	0.078	0.041	0.038	0.031	0.095	0.030	0.166	0.138
2008	0.066	0.044	0.034	0.039	0.080	0.034	0.178	0.153
2009	0.054	0.038	0.032	0.027	0.074	0.033	0.166	0.142
2010	0.048	0.033	0.027	0.023	0.073	0.033	0.161	0.132
2011	0.054	0.043	0.028	0.020	0.064	0.031	0.177	0.150
2012	0.065	0.044	0.026	0.025	0.060	0.027	0.212	0.182
2013	0.072	0.042	0.038	0.012	0.058	0.028	0.231	0.196
2014	0.082	0.044	0.034	0.020	0.061	0.031	0.208	0.182
2015	0.079	0.035	0.038	0.019	0.075	0.035	0.231	0.203
State Average	0.072	0.042	0.035	0.025	0.084	0.033	0.188	0.161

Note: To build the state averages we first calculated the share of the value of production (obtained by summing up the revenue from timber, milk and timber) per state and per year. Second, we multiply the technical change rate by these shares. Finally, we sum the observation within a state and a year. To build the region weighted average, we first obtained the share of the value of production per year (only). Second, we multiply the technical change rate by these shares. Finally, we sum the observations within a year.

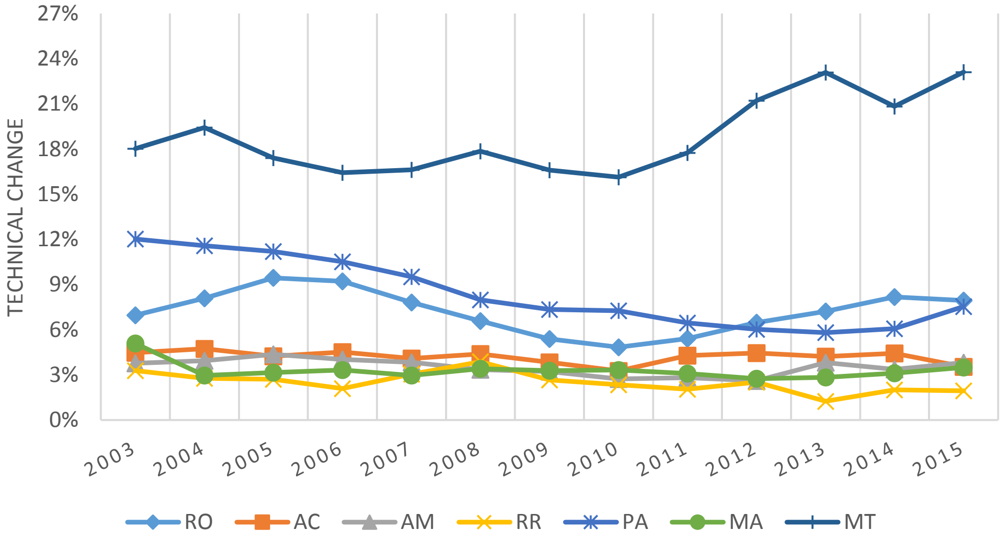

Figure B31 – Stochastic weighted state average technical change rates in the "arc of deforestation" in the northern region of Brazil.

Note: In this figure, we multiply Table C1 outcome by one hundred to obtain technical change in percentage. However, given our normalization of the data by the mean, this number should be interpreted only at the mean.

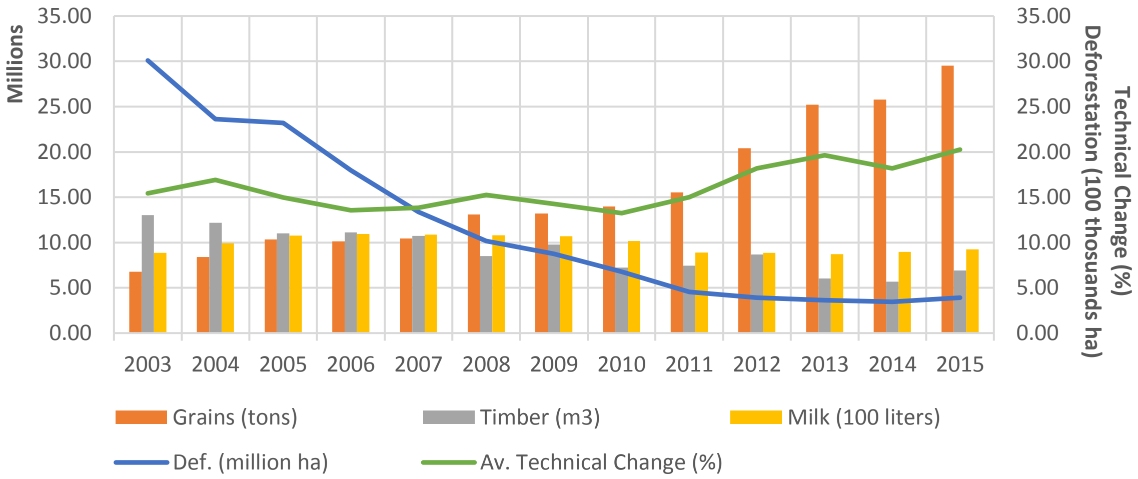

Figure B32 – Evolution of agricultural output, deforestation and the weighted average rate of technical change for the "arc of deforestation" in the northern region of Brazil.

Note: Grains (in tons), timber (in m^3) and milk (in 100 liters) vertical axis is displayed at left axis while deforestation (in 100 thousand hectares) and technical change (in percentage) on the axis at the right.

Appendix 3.C – Priority municipalities

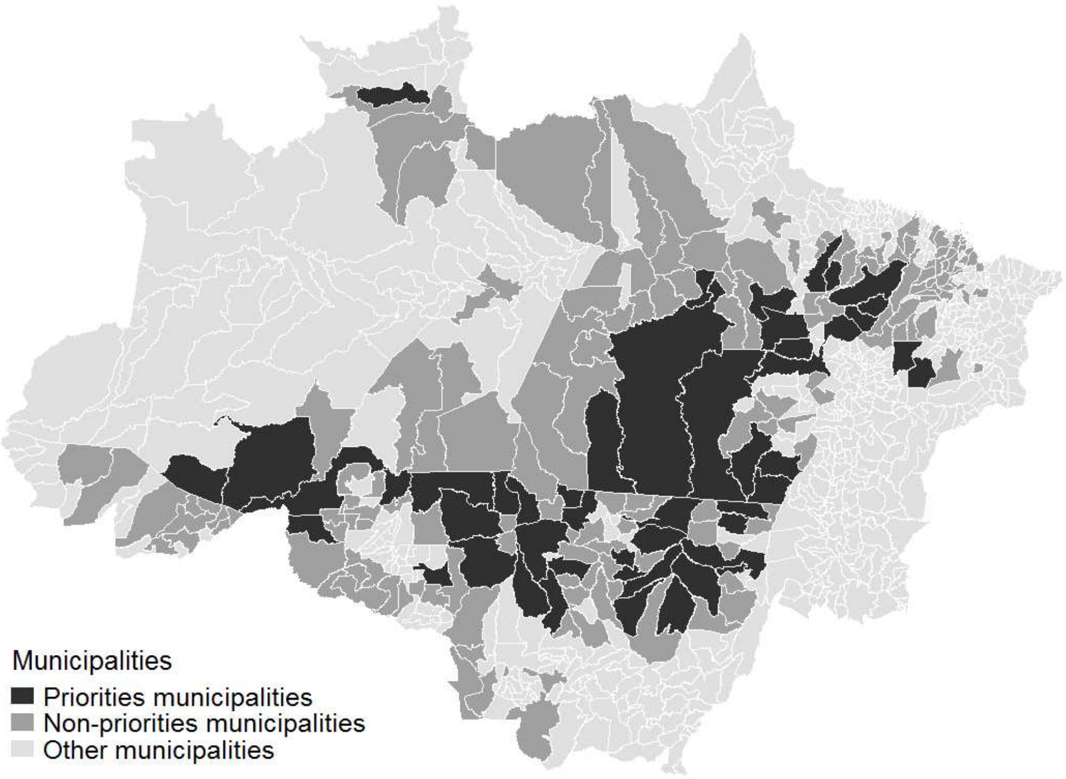

Figure 3.C1 – Priority municipalities in deforestation control defined by the Brazilian government.

Appendix I – Directional distance function properties description

The output distance function derives its properties from the output possibility set $P(x)$. These are (Färe *et al.*, 2005)

$$\vec{D}_o(x, y, b; g_y, -g_b) \geqq 0 \; if \; and \; only \; if \; (y, b) is \; an \; element \; of \; P(x) \qquad \text{(I 1.i)}$$

$$\vec{D}_o(x, y', b; g_y, -g_b) \geqq \vec{D}_o(x, y, b; g_y, -g_b) \; for \; (y', b) \leqq (y, b) \in P(x) \qquad \text{(I 1.ii)}$$

$$\vec{D}_o(x, y, b'; g_y, -g_b) \geqq \vec{D}_o(x, y, b; g_y, -g_b) \; for \; (y, b') \geqq (y, b) \in P(x) \qquad \text{(I 1.iii)}$$

$$\vec{D}_o(x, \theta y, \theta b; g_y, -g_b) \geqq 0 \; for \; (y, b) \in P(x) \; and \; 0 \leqq \theta \leqq 1 \qquad \text{(I 1.iv)}$$

$$\vec{D}_o(x, y, b; g_y, -g_b) \; is \; concave \; in \; (y, b) \qquad \text{(I 1.v)}$$

The directional distance function is non-negative (I 1.i) for feasible output vectors (***y*, *b***), and will take a value equal to zero when the unit is on the frontier. Desirable outputs are strongly disposable (I 1.ii), which means, that the directional distance function is non-increasing in desirable outputs for given inputs and undesirable outputs. The directional output distance function is non-decreasing in undesirable outputs (I 1.iii) and both desirable and undesirable outputs are jointly weakly disposable (I 1.iv). Properties (I 1.ii) and (I 1.iii) states the monotonicity condition of desirable and undesirable outputs, respectively, which means that the output distance does not increase as desirable outputs increase and as undesirable outputs decrease. The directional output distance function is concave in both types of outputs (I 1.v), which determines the sign of transformation elasticities. These properties can be seen in Figure 2 of chapter 1 and 3 and Figure 1 of Chapter2.

The null-jointness property states that no desirable output can be produced without generating undesirable output. Jointly with weak disposability of undesirable outputs, it implies that it is costly to reduce the undesirable output. This cost is measured in terms of foregone production of desirable outputs. It is represented by:

$$if \; (y, b) \in P(x) \; and \; b = 0 \; then \; y = 0 \; which \; implies \; \vec{D}_o(x, y, 0; g_y, -g_b) < 0 \qquad \text{(I 1.vi)}$$

which can be interpreted as a negative output directional distance, since the observation (y,0) is not in $P(x)$, as it would contradict property (I 1.i). It is represented in Figure 1 of chapter 2 by the origin of the production possibility frontier. The directional output distance function can be used as an efficiency measure. For firms on the production possibility frontier it takes a value of zero and for firms within it takes non-negative values.

Additionally, we impose the translation property[45], which corresponds to Shepard's (2015) homogeneity condition for the output distance function (Färe *et al.* 2006), and insures that the directional output distance function is homogeneous of degree one in outputs,

$$\vec{D}_o(\boldsymbol{x}, \boldsymbol{y} + \lambda g_y, \boldsymbol{b} - \lambda g_b; g_y, -g_b) = \vec{D}_o(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; g_y, -g_b) - \lambda, \quad \lambda \in \Re \qquad \text{(I 1vii)}$$

which states that increasing desirable outputs by λg_y while decreasing undesirable outputs by $-\lambda g_b$ is equivalent to subtracting the translation factor λ from the original directional distance function. The distance function obtained by increasing desirable outputs by βg_y while decreasing undesirable outputs by βg_b (Färe *et al.*, 2005), is the same as subtracting the translation factor α from the original directional distance function. We demonstrate this property below using a hypothetical case with one desirable outputs, one undesirable output, and one inputs. The subscripts identifying cross section units are dropped for sake of clarity. This section follows Hudgins and Primont (2007). First, we assume a quadratic directional output distance function

$$\vec{D}_o(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; g_y, -g_b) = \alpha_0 + \alpha_1 x_1 + \frac{1}{2}\alpha_{11}x_1x_1 + \beta_1 y_1 + \theta_1 b_1 + \frac{1}{2}\beta_{11}y_1y_1 + \frac{1}{2}\theta_{11}b_1b_1 + \delta_{11}x_1y_1$$
$$+\varphi_{11}x_1b_1 + \mu_{11}y_1b_1$$

Now, with respect to the translation property, we can re-write the left-hand side of the equation as

---

[45] Luenberger (1992) proves the translation property using a benefit function.

$$\begin{aligned}&= \alpha_0 + \alpha_1 x_1 + \frac{1}{2}\alpha_{11} x_1 x_1 + \beta_1(y_1 + \lambda) + \theta_1(b_1 - \lambda) + \frac{1}{2}\beta_{11}(y_1 + \lambda)(y_1 + \lambda) + \frac{1}{2}\theta_{11}(b_1 \\ &\qquad - \lambda)(b_1 - \lambda) + \delta_{11} x_1(y_1 + \lambda) + \varphi_{11} x_1(b_1 - \lambda) + \mu_{11}(y_1 + \lambda)(b_1 - \lambda)\end{aligned}$$

$$\begin{aligned}&= \alpha_0 + \alpha_1 x_1 + \frac{1}{2}\alpha_{11} x_1 x_1 + \lambda\beta_1 + \beta_1 y_1 + \frac{1}{2}\beta_{11} y_1 y_1 + \lambda\beta_{11} y_1 + \lambda\lambda\frac{1}{2}\beta_{11} + \theta_1 b_1 - \theta_1\lambda \\ &\qquad + \frac{1}{2}\theta_{11} b_1 b_1 - \theta_{11} b_1\lambda + \frac{1}{2}\theta_{11}\lambda\lambda + \delta_{11} x_1 y_1 + \lambda\delta_{11} x_1 + \varphi_{11} x_1 b_1 - \varphi_{11} x_1\lambda \\ &\qquad + \mu_{11} y_1 b_1 - \lambda\mu_{11} y_1 + \lambda\mu_{11} b_1 - \mu_{11}\lambda\lambda\end{aligned}$$

which can be rearranged by factorizing λ out

$$\begin{aligned}&\overbrace{= \alpha_0 + \alpha_1 x_1 + \frac{1}{2}\alpha_{11} x_1 x_1 + \beta_1 y_1 + \frac{1}{2}\beta_{11} y_1 y_1 + \theta_1 b_1 + \frac{1}{2}\theta_{11} b_1 b_1 + \delta_{11} x_1 y_1 + \mu_{11} y_1 b_1 + \varphi_{11} x_1 b_1}^{Part\ A} \\ &\underbrace{+\lambda\left[\beta_1 + \beta_{11} y_1 + \lambda\frac{1}{2}\beta_{11} + \theta_1 - \theta_{11} b_1 + \frac{1}{2}\theta_{11}\lambda + \delta_{11} x_1 - \varphi_{11} x_1 + -\mu_{11} y_1 + \mu_{11} b_1 - \mu_{11}\lambda\right]}_{Part\ B}\end{aligned}$$

Now, we can use of equation (I 1vii), which means equalizing to $\vec{D}_o(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; g_y, -g_b) - \lambda$, and cancelling some terms out, in specifically, the first row of the previous equations (*Part A*) we obtain:

$$\lambda\left[\beta_1 + \beta_{11} y_1 + \lambda\frac{1}{2}\beta_{11} - \theta_1 - \theta_{11} b_1 + \frac{1}{2}\theta_{11}\lambda + \delta_{11} x_1 - \varphi_{11} x_1 - \mu_{11} y_1 + \mu_{11} b_1 - \mu_{11}\lambda\right] = -\lambda$$

dividing both sides by λ, we obtain

$$\beta_1 + \beta_{11} y_1 + \lambda\frac{1}{2}\beta_{11} - \theta_1 - \theta_{11} b_1 + \frac{1}{2}\theta_{11}\lambda + \delta_{11} x_1 - \varphi_{11} x_1 - \mu_{11} y_1 + \mu_{11} b_1 - \mu_{11}\lambda = -1$$

differentiate with respect to x_1,

$$\delta_{11} - \varphi_{11} = 0$$

which is a simplified version of equation (7i) in the chapter 2. Now, differentiate with respect to y_1 and b_1,

$$\beta_{11} - \mu_{11} = 0$$
$$-\theta_{11} + \mu_{11} = 0$$

which is also represented in equation (7i) of chapter 2. These equations can be arranged as

$$\beta_{11} = \mu_{11} = \theta_{11}$$

plugging all these results we found that

$$\beta_1 + \beta_{11}y_1 + \lambda\frac{1}{2}\beta_{11} - \theta_1 - \beta_{11}b_1 + \frac{1}{2}\beta_{11}\lambda + \ \delta_{11}x_1 - \delta_{11}x_1 \ - \ \beta_{11}y_1 + \beta_{11}b_1 - \beta_{11}\lambda = -1$$

and several of these terms cancel out leading to

$$\beta_1 - \theta_1 = -1$$

which is also stated in Equation (7i) in chapter 2. These findings are applicable to a model with one desirable output and one undesirable output, except for the restriction in inputs given that in this hypothetical model we have only one input. Imposing the translation property in a model with more than one input would lead to a similar result, illustrated in chapter 2 and 3. We will continue to assume that we have only one input, one desirable output and one undesirable output. Let's assume that $b_1 = \lambda$ and that $g_y = g_b = 1$, re-arranging Equation (II 1i)

$$\vec{D}_o\left(\boldsymbol{x}, \boldsymbol{y} + \lambda g_y, \boldsymbol{b} - \lambda g_b; g_y, -g_b\right) + \lambda = \vec{D}_o\left(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; g_y, -g_b\right)$$

$$\begin{aligned}\vec{D}_o(\boldsymbol{x}, y_1 + b_1, b_1 - b_1; 1, -1) \\ &= \alpha_0 + \alpha_1 x_1 + \frac{1}{2}\alpha_{11}x_1x_1 + \beta_1(y_1 + b_1) + \theta_1(b_1 - b_1) + \frac{1}{2}\beta_{11}(y_1 + b_1)(y_1 + b_1) \\ &+ \frac{1}{2}\theta_{11}(b_1 - b_1)(b_1 - b_1) + \delta_{11}x_1(y_1 + b_1) + \varphi_{11}x_1(b_1 - b_1) + \mu_{11}(y_1 + b_1)(b_1 \\ &- b_1)\end{aligned}$$

which leads to

$$-b_1 = \ \alpha_0 + \alpha_1 x_1 + \frac{1}{2}\alpha_{11}x_1x_1 + \beta_1(y_1 + b_1) + \frac{1}{2}\beta_{11}(y_1 + b_1)(y_1 + b_1) + +\delta_{11}x_1(y_1 + b_1)$$

and it is estimated using different methodologies. This equation can be estimated using Corrected Ordinary Least Squares, Generalized Method of Moments, and Maximum Likelihood Estimation. It is important to notice that we also imposed symmetry. Hudgins and Primont (2007) asserts that the quadratic and the transcendental exponential are the only candidates to the application of the translation property.

A quadratic flexible functional form allows to evaluate global concavity, property (I 1.v), by analyzing the matrix of second order derivatives, the Hessian matrix. This matrix is composed only by parameters, different from other functional forms such as translog. Equation (I 2) displays this matrix for the directional distance function used in chapter 3

$$H = D^2\vec{D}_{o,i} = \begin{vmatrix} \theta_{11} & \mu_{11} & \mu_{21} & \mu_{31} \\ \mu_{11} & \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \mu_{21} & \gamma_{12} & \gamma_{22} & \gamma_{23} \\ \mu_{31} & \gamma_{13} & \gamma_{23} & \gamma_{33} \end{vmatrix} \tag{I 2}$$

where H denominates the Hessian matrix and $D^2\vec{D}_{o,i}$ defines it as the second order, D^2, derivatives of the directional output distance defined in Equation (5) of chapter 3. The reader has to bear in mind that some of these coefficients are recovered using the translation property [they are θ_{11}, μ_{11}, $\mu_{21}$and μ_{31}].

The Hessian matrix is concave if it is negative semidefinite (I 2i) and strictly concave if it is negative definite (I 2ii).

Appendix II – Stochastic Frontier: Theory and practice

This appendix was written based on Kumbhakar and Lovell (2000), Aigner, Lovell and Schmidt (1977), and Kumbhakar, Wang and Horncastle (2015). Stochastic frontier models were first presented simultaneously by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977). This methodology allows producers' output to be affect by random shocks outside their control and technical inefficiency. Production of y is obtained using J inputs x and affected by these two factors as

$$y_i = \beta_0 + \sum_{j=n}^{J} \beta_j x_{ji} + v_i - u_i \tag{II.1}$$

where β's are the parameter to be estimated, v_i is the two-side noise component and u_i is the non-negative technical inefficiency component. v_i is i.i.d symmetric distributed independently of u_i. Estimation of equation (II.1) under the assumption that both v_i and u_i are independently distributed from x_{ji} leads to consistent estimation of the β_j's but not of β_0 given that $E(\varepsilon) = E(v_i - u_i) = -E(u_i)$. A simple test of skewness of the distribution of ε allows to infer whether producers' inefficiency is different from zero. $u_i = 0$ implies that all parameter of Equation (II.1) estimated by Ordinary Least Square (OLS) will be consistent while $u_i > 0$ implies that the constant term, β_0, has to be corrected.

Although estimation of equation (II.1) using OLS does not provide directly information on technical inefficiency it does not impose any distribution to this component. Aigner, Lovell and Schmidt (1977) proposed to estimate Equation (II.1) using Maximum Likelihood estimation (MLE) approach imposing a half-normal distribution on the inefficiency component, u_i. This estimation does not lead to non-consistency of the constant term, β_0. The following assumptions are required

$$v_i \sim iid\ N^{+}(0, \sigma_v^2) \tag{II.2i}$$

$$u_i \sim iid\ N^{+}(0, \sigma_u^2) \tag{II.2ii}$$

v_i and u_i are distributed independently of each other and of the regressors (II.2iii)

Given these distributions, the Log Likelihood function for a sample of *N* producers is defined as

$$lnL = \frac{N}{2} * ln2 - \frac{N}{2} * ln\pi - N * ln\sigma + \sum_{N} ln\Phi\left(-\frac{\varepsilon_i \lambda}{\sigma}\right) - \frac{1}{2\sigma^2}\sum_{N} \varepsilon_i^2 \tag{II.8}$$

where $\sigma = (\sigma_v^2 + \sigma_u^2)^{1/2}$, $\lambda = \sigma_u/\sigma_v$ and Φ is the standard normal cumulative distribution. Kumbhakar *et al.* (2000) indicates that as $\lambda \to 0$ the one-sided error component dominates the determination of ε while if $\lambda \to +\infty$ the technical inefficiency error component dominates the determination of ε. The first case indicates the OLS could be used while in the second case producers' output is not affected by random shocks (a linear programming would be suitable). Both σ and λ are jointly estimated with the β's. Technical inefficiencies are estimated as proposed by Jondrow, Lovell and Schmidt (1982), by taking the mean of the conditional distribution of *u* given ε

$$f(u|\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma_*} exp\left\{-\frac{(u-\mu_*)^2}{2\sigma_*^2}\right\} \Big/ \left[1 - \Phi\left(-\frac{\mu_*}{\sigma_*}\right)\right] \tag{II.9i}$$

$$E(u|\varepsilon) = \mu_{*i} + \sigma_* \left[\frac{\Phi(-\mu_{*i}/\sigma_*)}{1 - \Phi(-\mu_{*i}/\sigma_*)}\right] \tag{II.9ii}$$

where $\sigma_* = \sigma_v \sigma_u / \sigma$ and $\mu_* = -\varepsilon\sigma_u^2/\sigma^2$.

Additionally, in chapter 2 we assume that *u* is heteroskedastic. It implies that municipality-specific factors influence technical efficiency. It requires substituting (II.2ii) by $u_i \sim iid\ N^+(0, \sigma_{ui}^2)$ with $\sigma_{ui}^2 = g_2(z_i; \delta_2)$ where the municipality-specific factors are z_i and δ_2 the estimated parameters. The Log Likelihood function become than

$$lnL = \frac{N}{2} ln2 - \frac{N}{2} ln\pi - N\ln[g_2(z_i;\delta_2) + \sigma_v^2] + \sum_{N} ln\Phi\left(-\frac{\varepsilon_i\lambda_i}{\sigma_i}\right) - \frac{1}{2}\sum_{N}\frac{\varepsilon_i^2}{\sigma_i^2} \tag{II.10}$$

where $\sigma_i^2 = \sigma_v^2 + \sigma_{ui}^2 = \sigma_v^2 + g_2(z_i;\delta_2)$ and $\lambda = \sqrt{g_2(z_i;\delta_2)}/\sigma_v$. After estimation, values of σ_v^2 and σ_{ui}^2 can be substitute in (II.9ii) to obtain technical inefficiencies.

In chapter 2 and chapter 3 we use both[46] COLS and MLE approaches to estimate the directional distance function. However, the translation property imposes a few changes in Equation (II.1). In our estimation, $y_i = -b_i$ where b_i represents the undesirable output (deforestation in both cases), x_{ji} are composed by all linear and quadratic terms formed by output and inputs, and u_i represents the distance, which is a measure of technical inefficiency. A deeper discussion of these methodologies can be found in Kumbhakar *et al.* (2000).

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[46] In chapter 2 we even used Generalized Method of Moments as a robustness check to test whether endogeneity plays a role on shadow price estimates. In Both chapter 2 and 3 we also estimate the directional distance function using a non-stochastic approach, Linear Programming. These estimations are not presented in here because they have led to similar results.